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Final Report

LAYER COEFFICIENTS IN TERMS OF PERFORMANCE AND MIXTURE CHARACTERISTICS

Brian Coree

Thomas D. White

LAYER COEFFICIENTS IN TERMS OF PERFORMANCE
AND MIXTURE CHARACTERISTICS
FINAL REPORT

TO: H. L. Michael, Director   September 5, 1988
       Joint Highway Research Project

                                          Project: C-36-55H

FROM: T. D. White, Research Engineer
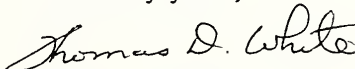   Joint Highway Research Project   File: 2-12-88

Attached is the Final Report on the HPR Part II Study titled, "Layer
Coefficients in Terms of Performance and Mixture Characteristics." This
report documents a study that examined in detail the origination of the AASHTO
layer coefficient. As a result, an analysis procedure was developed that
estimates the layer coefficient for asphalt mixtures used in Indiana. The
method is general and is applicable to most asphalt mixtures. Asphalt
properties, loading, rate, aggregate gradation characteristics and climatic
conditions are generally accounted for in the procedure.

Conditions and properties of materials at the road test were developed on a
distributive basis. A corresponding distributive analysis was conducted of
Indiana asphalt mixtures with respect to climatic regions in the state.
Previously the AASHO Road Test has not been examined on the basis of the layer
coefficient variation. Such an analysis provides a more logical evaluation of
the layer coefficient to be used in pavement design.

This report is forwarded to IDOH and FHWA in fulfillment of the objectives of
the study.

                            Sincerely yours,

                            Thomas D. White
                            Research Engineer

TDW/cab

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16. Abstract

A set of AASHTO Layer Coefficients has been derived for the ten (10) bituminous mixtures currently specified by the Indiana Department of Highways (IDOH). This project was initiated by IDOH in response to a perceived need to optimize material usage through the design process.

Layer Coefficients previously derived by other Agencies were found to be inadequate for a variety of reasons. Indeed, Layer Coefficients as derived at the AASHO Road Test were shown to be flawed in concept (Appendix A).

Recognizing that bituminous materials are very sensitive to temperature and time of loading, a probabilistic approach was used to explicitly account for the range and variety of environmental and traffic conditions encountered in Indiana. Equally, in-place bituminous mixtures represent sample values of specification envelopes, or tolerances: the probable range of mixture parameters was used in the analysis to derive Layer Coefficient Distributions rather than unique, deterministic values.

Two powerful methods were used in the analysis: the van der Poel/Ullidtz/Bonnaure et al. method of predicting bituminous material stiffness, S_b, and the Rosenblueth Point Estimate Method for dealing with variable distributions rather than mean, or expected, values. It is believed that the resulting Layer Coefficients are more realistic and represent the true range of behavior observed in practice.

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LAYER COEFFICIENTS IN TERMS OF PERFORMANCE
AND MIXTURE CHARACTERISTICS

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and the

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ABSTRACT

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Layer coefficients previously derived by other Agencies were found to be inadequate for a variety of reasons. Indeed, layer coefficients as derived at the AASHO Road Test were shown to be flawed in concept (Appendix A).

Recognizing that bituminous materials are sensitive to temperature and time of loading, a probabilistic approach was used to explicitly account for the range and variety of environmental and traffic conditions encountered in Indiana. Equally, in-place bituminous mixtures represent sample values of specification envelopes, or tolerances: the probable range of mixture parameters was used in the analysis to derive layer coefficient distributions rather than unique, deterministic values.

Two powerful methods were used in the analysis: the van der Poel/ Bonnaure et al. method of predicting bituminous material stiffness, S_b, and the Rosenblueth Point Estimate Method for dealing with variable distributions rather than mean, or expected values. It is believed that the resulting layer coefficients are more realistic and represent the true range of behavior observed in practice.

THE LAYER COEFFICIENT: ITS ORIGIN AND MEANING.

The layer coefficient has its origins in the AASHO Road Test (1958-61). The 1972 edition of the AASHTO Interim Guide for Design of Pavement Structures (1) states:

"2.4.4 — Layer Coefficients. As mentioned in the previous section, a coefficient must be assigned to each material used in the pavement structure in order to convert structural number to actual thickness. This layer coefficient expresses the empirical relationship between SN and thickness, and is a measure of the relative ability of the material to function as a structural component of the pavement."

Consequently, the layer coefficient is an empirical measure of contribution to structural performance. The overall structural measure (capacity, adequacy), the Structural Number, SN, of a pavement is given in the same publication as:

"2.4.3 — Structural Number. The solution of the design equation in this guide is in terms of a structural number (SN). The structural number is an abstract number expressing the structural strength of pavement required for a given combination of soil support value, total equivalent 18-kip (80kN) single axle loads, terminal serviceability index, and regional factor."

As a result, the layer coefficient is an empirical component of an abstract number, both of which are inferred to be measures of strength. These definitions from the 1972 Interim Guide provide

little guidance as to the means of measurement of either of these parameters; indeed, they provide little substance as to their significance.

In order to provide a more precise definition, recourse was made to the full report of the AASHO Road Test (the several volumes of HRB Special Report 61 (2)), in which the complete methodology and analysis is given in detail. Reference was also made to a paper by Carey and Irick (3), which provided the basis of the serviceability measurement at the Road Test.

The AASHO Road Test Analysis

The following analysis is limited to the flexible sections of the Road Test since the layer coefficient concept was not used in the analysis of the rigid pavement sections.

In HRB 61E, the overall model of pavement performance is given as:

$$p = c_o - (c_o - c_1) \left[\frac{W}{\rho}\right]^\beta \tag{1}$$

where
- p = the serviceability trend value,
- c_o = the initial serviceability trend value.
- c_1 = the serviceability level at which a test section was considered out-of-service.
- W = the accumulated axle load applications at the time when p was observed.
- ρ, β = functions of design (thickness) and load.

Conventionally, equation (1) is rearranged in the following, more familiar form:

$$\log_{10}(W) = \log_{10}(\rho) + \frac{G}{\beta} \tag{2}$$

where
$$G = \log_{10}\left[\frac{p_o - p}{p_o - p_t}\right] = \log_{10}\left[\frac{c_o - p}{c_o - c_1}\right]$$

wherein c_i and p_t are synonymous, (in the original formulation, c referred to condition, and p to present serviceability).

Two variables in equation (2) are obtained by direct measurement, namely W, the number of axle repetitions, and p, the serviceability. p_o and p_t were arbitrarily assigned values of 4.2 and 1.5 respectively at the Road Test).

Present serviceability index, p, is a composite parameter, measured by direct evaluation of a select set of distress types (roughness, cracking, patching and rut depth). The relationship which relates serviceability to the direct measurements is the result of a regression analysis of the observation of these distress types to subjective ratings of pavement serviceability as perceived by panels of both expert and non-expert road users.

The rating of these panels, known as the present serviceability rating (PSR), is purely subjective. It is not a structural rating, but more truly a "seat-of-the-pants" ranking of the perceived ride quality of a pavement. Specifically, it is the aggregated perception of ride quality given by panels in the period 1957-1960. There is no assurance that the results could be duplicated at any later time. Road user perception is not expected to be constant. Consequently, the present serviceability index, p, is based on perceptions and relationships founded in the late fifties/early sixties.

It is clear that the measurement of the serviceability index provides an estimate of the ride quality of a pavement. It is not a measure of strength.

At the Road Test, there were many concurrent experiments, but the one which is of concern herein is the Main Factorial Experiment. The Main Factorial Experiment consisted of 332 pavement sections, each 100 feet in length. Forty eight (48) sections in loop 1 of the Main Factorial Experiment were not subjected to traffic during the Test, and served as untrafficked reference sections. Two hundred

and eighty four (284) sections were trafficked and included in the final analysis. Forty four (44) duplicated or replicate sections were included in the experiment to provide a measure of statistical robustness to the analysis.

The pavement sections were constructed of varying thicknesses of three pavement materials; a bituminous surfacing, a crushed dolomite limestone base, and an uncrushed natural sand-gravel subbase. The subgrade consisted of a 3 ft. thickness of an A-6 (CL) soil placed at slightly above optimum moisture content.

Each loop consisted of two lanes. Each lane was dedicated to traffic of specific weight and axle configuration. Thus, the measured axle repetitions, W, on each section is wholly specified by two further parameters; L_1, the axle weight (in kips), and L_2, axle type (single or tandem).

Under this scenario, the present serviceability index, p, was measured on each section at intervals of two weeks. Thus, for each section, a history of traffic (W, L_1, L_2) and present serviceability index (p) was created.

It is reported in HRB SR61E that the serviceability data (i.e., the present serviceability index) for each section was "smoothed". Details of this are not given, but are assumed to be arithmetic moving averages. Two records of traffic (W) were kept: unweighted traffic (W) was the true repetitions of axle (L_1, L_2) and was unadjusted, while weighted traffic W_t was adjusted (2) in an attempt to take into account seasonal variations in damage.

Taking into account the assumed initial serviceability ($p_o = c_o = 4.2$) and terminal serviceability ($p_t = c_t = 1.5$) conditions, equation [2] may be re-written:

$$\log_{10}(W) = \log_{10}(\rho) + \frac{\log\left(\frac{4.2 - p}{2.7}\right)}{\beta} = \log_{10}(\rho) + \frac{G}{\beta} \qquad (3)$$

It may be seen that this relationship is linear in $\log_{10}(W)$ vs. G.

The parameters $\log_{10}(\rho)$ and $1/\beta$ represent $\log_{10}(W)$ at $p = 1.5$ and the slope of the linear relationship respectively.

Since the W vs. p history for each section was known by direct measurement, simple linear regression analysis provides estimates of ρ and β for each section. These parameters are defined as functions of design and load, and as such are defined 'a priori' as:

$$\rho = A_0 (D + a_4)^{A_1} (L_1 + L_1)^{A_2} (L_1)^{A_3} \qquad (4a)$$

$$\beta = \beta_0 + B_0 (D + a_4)^{B_1} (L_1 + L_2)^{B_2} (L_2)^{B_3} \qquad (4b)$$

where L_1, L_2 are the axle weight (kips) and type,(1 = single, 2 = tandem)

          D is a composite parameter of pavement design, the thickness index,

          A_i, B_i are regression coefficients, and

          β_0 is provided such that the denominator on the right hand-side of equation (3) may not become zero.

The thickness index, D, was further defined as the linear combination of layer thicknesses, so that $D = a_1 t_1 + a_2 t_2 + a_3 t_3 + a_4$, where the t_i denote the layer thicknesses in inches. The coefficients a_4 and β_0 were arbitrarily assigned the values 1.0 and 0.4 respectively, (1.0 is a *nice* number, and 0.4 is "any small, non-negative number" sufficient to prevent the denominator in equation (3) from becoming zero (2)).

The previously estimated values of ρ and β for each section were then used in a regression analysis to determine the coefficients A_{0-3} and B_{0-3}. This regression was initially undertaken with the coefficients a_i set to unity such that $D = \sum t_i$. Subsequently, a variant regression was undertaken letting the values of the coefficients a_i float. This latter regression yielded $D = SN =$

$\sum(a_i.t_i)$, where the thickness index D was termed the structural number SN, and the coefficients a_i the layer coefficients.

Thus, it may be seen that the original definition of layer coefficient (and structural number) gives no more significance than secondary regression coefficients (ρ and β being the primary regression coefficients).

Two measured variables (W and p), and five parameters (L_1, L_2, t_1, t_2, and t_3) were used in the analysis. W, L_1 and L_2 are purely measures of load (frequency (number of repetitions), magnitude and type): the t_i are measures of pavement geometry and the serviceability index p is a measure of the perceived pavement ride quality.

NO MEASURE OF STRENGTH WAS USED IN THE ROAD TEST ANALYSIS

STRUCTURAL LAYER COEFFICIENT DERIVATION

While the layer coefficients, a_i, have been described as "structural" from the original report SR61E up to and including the 1986 issue of the AASHTO Guide (4), this description is not supported by an analysis of the original formulation. More rational and accurate would be to ascribe to the layer coefficient the attributes of "contribution to resistance to functional loss", in other words to refer to it as the "functional layer coefficient".

Reference 28 describes a powerful reasoning as to why the original Road Test analysis is flawed and why discussion of the true significance of the layer coefficient is almost meaningless.

Conventionally, layer coefficients have been assumed to be indicators of strength. Much effort has been expended in measuring or estimating layer coefficients for new materials (or materials other than those used at the Road Test).

Two predominant methods have been used to provide estimates of layer coefficients:

1   Direct: The layer coefficient, a_i, is related directly to a more conventional strength parameter, for example:

$$a_i = A \cdot M_R^{\ B} \qquad \text{(AASHTO Guide 1986, Fig. 2.5)} \qquad (5)$$

where   A,B are experimentally derived constants, and
          M_R is the Resilient Modulus (AASHTO T274)

This type of definition is absolute, where the layer coefficient is uniquely determined for the material by reference to the chosen strength parameter.

2   Relative: The unknown layer coefficient, a_i, is found relative to the "known" coefficient, a_{ref}, of a different material through the ratio of some strength parameter, for example:

$$a_i = a_{ref} \cdot \left(\frac{M_{R_i}}{M_{R_{ref}}} \right)^B \tag{6}$$

This type of relationship will be shown to be analogous to Odemark's equivalent stiffness hypothesis (5). (M_R and B have the same significance as in [5] above)

Since the direct type of layer coefficient measurement must in itself assume equivalence at some point in the relationship, the two methods may be considered identical, since for example:

$$a_i = a_{ref} \cdot \left(\frac{M_{R_i}}{M_{R_{ref}}} \right)^B = \left[\frac{a_{ref}}{\left(M_{R_{ref}} \right)^B} \right] \cdot M_{R_i}^B = A_i \cdot M_{R_i}^B \tag{7}$$

since a_{ref} and $M_{R_{ref}}$ are constant in this relationship.

Using this concept, structural or strength properties may be used to estimate layer coefficients so long as the reference material is defined both by its layer coefficient and the required strength parameter.

This is in its essentials the method advocated by the 1986 AASHTO Guide, where the Resilient Modulus, M_R, is the preferred strength parameter. In the past, parameters such as the California Bearing Ratio (AASHTO T193) and Marshall Stability (ASTM D1559) have been used. Recent emphasis on a more fundamental measure of material strength has led to the proposed use of Resilient Modulus (M_R) in preference to the more empirically derived measures used previously.

If this concept is accepted, then the problem of estimating layer coefficients is considerably simplified. However, the question remains as to what magnitude of the strength parameter should be taken for the reference material, i.e., under what conditions (temperature, stress intensity or frequency, etc.) should the layer coefficient – reference material relationship apply? This is a particularly sensitive question in bituminous materials. For example, the resilient modulus may vary by orders of magnitude depending upon temperature, frequency of loading and mixture parameters.

Considering temperature alone, it will be appreciated that the resilient modulus of a bituminous material in August will be substantially different from that in January. Not only will these values be different, but the ratio:

$$\left(\frac{M_{R_{bituminous}}}{M_{R_{granular}}}\right)$$

(i.e., the relative strengths of the bituminous surfacing and the granular base course) will vary continuously throughout the year (or climatic cycle).

In this light, the layer coefficients obtained at the Road Test are seen to be time-averaged values. The fact that the Road Test layer coefficients are not unique deterministic values is well demonstrated in Table 9 of HRB SR61E where the layer coefficients for weighted traffic are seen to range as shown in Table 1.

Minimum	Coefficient	Maximum
0.33	a_1	0.78
0.12	a_2	0.23
0.07	a_3	0.12

Table 1. AASHO Road Test layer coefficients.
Range of values reported in HRB SR 61E (Table 9)
A similar set of results was obtained from a detailed analysis of

the raw AASHO data which provided the basis of the distributions
shown graphically in Figure 1. The distributions in this figure
were constrained to be non-negative, since it is intuitively
unlikely that an *increased* thickness of a pavement layer would lead
to a *lesser* strength.

Use of this distribution involves application of the Odemark
equivalent stiffness hypothesis (5) (see following section). As a
result, the following relationship may be stated:

$$a_i = a_{ref} \left[\frac{M_{R_i}}{M_{R_{ref}}} \right]^{\frac{1}{3}} \tag{8}$$

where a_i and M_{R_i} denote the layer coefficient and Resilient Modulus
of the material (mixture) in the ith layer, respectively.

Recognizing that the layer coefficient of the reference material,
a_{ref} is given, not by a *single* value, but by the *distribution* for
the bituminous surfacing material shown in Figure 1, the
distribution of values of the layer coefficient of the unknown
material is defined if the distributions of the resilient moduli of
both the reference material and the "unknown" material are known.
The only reference for bituminous material is that of the surfacing
material used at the AASHO Road Test. This approach was used in
this project, with the difference that "mixture stiffness" was used
rather than resilient modulus.

The question arises as to which value should be used for the mixture
stiffness S_m. Should the summer value be used, when, due to the
thermal inertia of the pavement materials, the bituminous material
is in its "weakest" state, or should the spring values be used when
the bituminous materials are still relatively "strong" compared to
the weakened state of the subgrade and thus more prone to cracking?
This question becomes more complicated when comparing mixtures under
different climatic environments; how may a bituminous material in
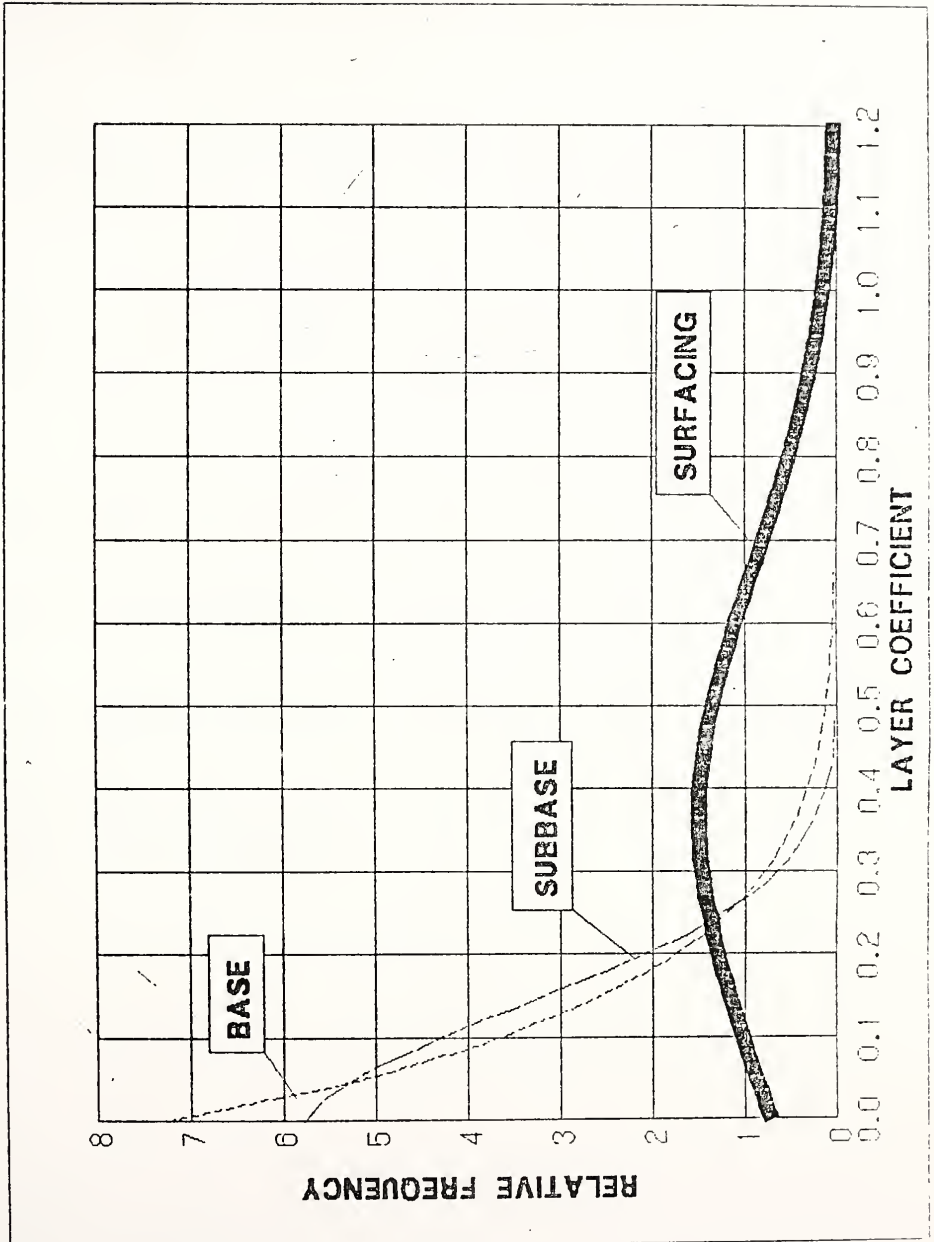
Figure 1. AASHO Road Test - Layer Coefficient Distributions

southern Texas (no freezing) be compared to the reference material (Ottawa, Illinois) where freeze/thaw conditions are significant?

Traditional mathematical methodologies which might be used for such analysis are complicated and require detailed knowledge of high-power derivatives of each of the variables. Newer, recently developed methodologies permit the comparison of material behavior over the full range of expected conditions with only a knowledge of the first three statistical moments and of any active correlation between variables. These techniques release us from the need to arbitrarily normalize material properties to some artificial state (e.g. typically 70° F).

A further benefit of these methods is that a lack of precise knowledge may be incorporated with advantage into the computations, thus making maximum use of minimal information. Of particular interest is the Rosenblueth point estimate method (6), which will be used in the development of this report.

Using the Odemark equivalent stiffness method, the Rosenblueth point estimate method and a means for estimating mixture resilient moduli (or mixture stiffness), the distribution of layer coefficients may be determined for any given bituminous mixture under any climatic conditions.

ODEMARK EQUIVALENT STIFFNESS CONCEPT

In 1949, N. Odemark at the Swedish Highway Institute developed an semi-empirical method for comparing different structures. While the method is semi-empirical in fact, it has its basis in structural engineering.

Consider a simply supported beam of length L, of depth t_1, and of unit width. From the theory of strength of materials, it is known that the stiffness is given by:

$$S_1 = \frac{1}{E_1 I_1} = \frac{12}{E_1 t_1^3}$$ (9)

where E_1 is the elastic (Young's) modulus of the beam
         material,

         I_1 is the second moment of area of the beam
         cross-section about its neutral axis.

Consider a beam of a different material, E_2, depth t_2, also of unit width, acting under the same load, then its stiffness will be:

$$S_2 = \frac{12}{E_2 t_2^3}$$ (10)

By requiring that the two beams have the same (or equivalent) stiffness, then:

$$S_{1=2} = \frac{12}{E_1 t_1^3} = \frac{12}{E_2 t_2^3}$$ (11)

which may be re-arranged, thus:

$$t_2 = t_1 \cdot \left(\frac{E_1}{E_2}\right)^{1/3} \tag{12}$$

This is the Odemark equivalent stiffness concept, which re-stated says that t_1 inches of material of modulus E_1 has the same stiffness as t_2 inches of a material of modulus E_2, where t_2 is given by:

$$t_1 \cdot \left(\frac{E_1}{E_2}\right)^{1/3}$$

If an n-layer structure (t_i, E_i, i=1,n) is considered, repeated application of the Odemark principle results in:

a)   The first layer (E_1, t_1) may be transformed to an equivalent thickness of the second layer material (E_2), thus:

$$t_{1,2} = t_1 \cdot \left(\frac{E_1}{E_2}\right)^{1/3} \tag{13}$$

Conceptually, the upper layer now consists of ($t_2 + t_{1,2}$) inches of E_2 material.

b)   This composite layer may subsequently be transformed into an equivalent thickness of the next layer (E_3, t_3):

$$t_{1,2,3} = (t_{1,2} + t_2) \cdot \left(\frac{E_2}{E_3}\right)^{1/3} = t_1 \cdot \left(\frac{E_1}{E_2}\right)^{1/3} \cdot \left(\frac{E_2}{E_3}\right)^{1/3} + t_2 \cdot \left(\frac{E_2}{E_3}\right)^{1/3} \tag{14a}$$

or,

$$t_{1,2,3} = t_1 \cdot \left(\frac{E_1}{E_3}\right)^{1/3} + t_2 \cdot \left(\frac{E_2}{E_3}\right)^{1/3} \tag{14b}$$

c)   Repeated application of this principle yields:

$$t_n = t_1 \left(\frac{E_1}{E_n}\right)^{1/3} + t_2 \left(\frac{E_2}{E_n}\right)^{1/3} + t_3 \left(\frac{E_3}{E_n}\right)^{1/3} + .. + t_{n-1} \cdot \left(\frac{E_{n-1}}{E_n}\right)^{1/3} \qquad (15)$$

where t_n inches of material E_n is equivalent (in stiffness) to the original n-layer pavement. It should be immediately apparent that the shape, or form, of the above relationship is identical to the structural number relationship, i.e.:

$$SN = a_1 t_1 + a_2 t_2 + a_3 t_3 \qquad (16)$$

where SN is equal to the equivalent thickness (which is the general definition of the Structural Number); a_i, the layer coefficient, is equal to the ratio of the layer modulus to that of a reference material raised to the one-third power.

By extending these relationships, the following assumptions can be made:

$$a_{idoh} = \left(\frac{E_{idoh}}{E^*}\right)^{1/3} \quad \text{and} \quad a_{aasho} = \left(\frac{E_{aasho}}{E^*}\right)^{1/3} \qquad (17)$$

where:

a_{idoh} The desired layer coefficient for an IDOH asphalt mixture,

a_{aasho} The 'measured' AASHO Road Test asphalt layer coefficient,

E^* A reference modulus,

E_{aasho} The modulus of the AASHO Road Test asphalt mixture.

and by division:

$$\frac{a_{idoh}}{a_{aasho}} = \left(\frac{E_{idoh}}{E_{aasho}}\right)^{1/3} \quad \text{or} \quad a_{idoh} = a_{aasho} \left(\frac{E_{idoh}}{E_{aasho}}\right)^{1/3} \qquad (18)$$

This relationship provides the basis of the method used in this

report. Since the layer coefficients (and their distributions) are known for the AASHO Road Test (as indeed are all other pertinent mixture and performance data), it requires only a knowledge of the parallel parameters of the IDOH mixtures to estimate their layer coefficients.

STIFFNESS MODEL FOR BITUMINOUS MIXTURES

In the above development of the Odemark equivalent stiffness method, use was made of Young's elastic modulus, E. In the remainder of this report, use will be made of binder stiffness, S_b, and mixture stiffness, S_m. The reasons for this are two-fold.   Bituminous materials are inherently visco-elastic, and as such their stress-strain behavior is both non-linear and influenced greatly by temperature and time (or frequency) of loading.   While it is possible to measure Young's modulus in bituminous materials, there does not exist a good method to relate mixture parameters (binder content, void content, etc.) to measured results.

Making use of binder and mixture stiffnesses, which are analogous to elastic moduli, permits use of existing models relating engineering mixture properties to stiffness, and through use of a Creep test, allows laboratory verification of the estimates.

Van der Poel(7) showed that the properties of bituminous mixtures are primarily influenced by the properties of the bituminous binder and by the volume concentration of the mineral aggregate within the mixture. The influence of these components on the properties of mixture models has been confirmed by Allen(8).

A nomograph which yields the behavioral properties of bituminous binders was published by van der Poel. With this nomograph, the binder stiffness S_b, may be estimated through a knowledge of the binder penetration (ASTM D5), binder softening point (ASTM D36), penetration index (10,11), frequency (or duration) of loading and temperature.

The accuracy of binder stiffness S_b, estimated by this method compared to laboratory results is reported by Pell(9) to be within a

factor of two. Granted that this level of accuracy is poor compared
to strength measurements of other more conventional engineering
materials, it nonetheless provides a good measure of the material
property and is widely accepted and used within the industry. At
the same time the accuracy of results obtained from laboratory
tests, particularly for bituminous materials, is hardly precise, and
may or may not truly reflect the performance of such materials in
service.

Ullidtz (9), provided a regression equation based upon the van der
Poel nomograph which permits the estimation of binder stiffness
within 10 percent of the nomograph values.

$$S_b \text{ (MPa)} = 1.157\times10^{-7} \ t^{-0.368} \ e^{-PI} \ (T_{RB} - T)^5 \tag{19}$$

where S_b = the binder stiffness (MPa)

      t = the time of loading (seconds)

      PI = the Penetration Index (10,11)

      T_{RB} = the ASTM (D36) Ring and Ball Softening Point ($^\circ$C)

      T = the ambient, or material test temperature ($^\circ$C)

Ullidtz reports that the above equation yields a ±10 percent
correspondence with the van der Poel nomograph when the variables
are within the limits given in Table 2. If the allowable
temperature range (T_{RB} - T) is increased to 15°C to 75°C, then the
corresponding accuracy is ±20 percent.

Minimum	Variable	Maximum
0.01	t, sec	0.20
20°C	(T -T)	60°C
-1	PI	+1

Table 2. Validity Limits on Ullidtz Equation

Effects of loading are incorporated in the above relation by t,
being the time of loading or the inverse of the frequency of

loading. The frequency (f=1/t) provides a good approximation to the vehicle speed (km/h). Ambient climatic conditions are covered by the temperature, $T^{\circ}C$. Material properties of the binder are represented by the penetration index, PI, and the softening point temperature, T_{RB}.

Bituminous material properties used in road construction are not constant during the life of the pavement. Effects such as heat during mixture production, and long term oxidation of the binder in place can affect the material's properties. While changes in property can not be accurately predicted, the methods of van der Poel, Ullidtz and others have provided the means whereby the in-service (or recovered) properties of bituminous materials may be reasonably estimated from initial properties.

An estimate may be made of the recovered binder penetration, Pen_r, from a knowledge of the initial penetration, Pen_i (ASTM D5):

$$Pen_r = 0.65 \ Pen_i \qquad (20)$$

the recovered T_{RB} softening point (ASTM D36) from:

$$T_{RB} = 99.13 - 26.35 \ \log_{10}(Pen_r) \qquad (21)$$

and the recovered penetration index from:

$$PI_r = \left[\frac{27 \ \log_{10}(Pen_r) - 31.2}{76.35 \ \log_{10}(Pen_r) - 219.27} \right] \qquad (22)$$

While these relations (developed by Ullidtz (9)) are now frequently used and accepted by the paving industry (although more commonly in Europe), no published statement of accuracy or range of validity has been found. However, applying these results to a wide range of tests conducted by the Bureau of Public Roads (BPR) (12, 13), the overall agreement was found to be excellent ($r^2 > 0.95$ in all cases). The BPR tests covered penetration-grade binders ranging from 60-70 penetration to 150-200 penetration, produced variously by vacuum distillation, steam distillation, blowing (oxidation), blending,

propane fractionation, fluxing, and combinations of these processes, from crudes obtained from throughout the USA and overseas.

The binder stiffness, S_b, is only one component of the behavior of the mixture. Heukelom and Klomp[14] developed a procedure to combine the aggregate volume concentration (volume of aggregate as a percentage of the total volume) and binder stiffness to estimate the mixture stiffness, S_m. However, the Heukelom and Klomp procedure is based on a single air voids content (3 percent) and a restricted range of mixture types. Other works by Samier and Bazin[15], Verstraeten[16] and others have provided alternate relationships, but also limited in application, usually to single mixture types.

In 1977, Bonnaure et al.[17], conducted extensive laboratory experiments using both laboratory prepared specimens and cores recovered from in-service pavements. The range of mixture types was large, five surfacing mixtures (two asphaltic concretes, a German Gussasfalt, a British Hot Rolled Asphalt and a British Open-graded mix), five base-course mixtures (two coarse asphaltic concretes, a Dutch gravel-sand asphalt, and two French bitumen-stabilized sands), an asphalt grouting mixture and a filler/bitumen mastic. The binder contents ranged from 4 to 24 percent by weight of aggregate, and the voids from 0 to 32 percent.. They proposed a methodology which estimates the mixture stiffness, S_m, valid for a wide range of mixture types and loading conditions within the accuracy of the van der Poel nomograph.

The Bonnaure et al. method requires the computation of four mixture parameters, each of which was found to have an influence upon the mixture stiffness.

$$s_z = 10.82 - 1.342 \left(\frac{100 - V_a}{V_a + V_b} \right) \tag{23a}$$

$$s_y = 8.0 + 5.68 \times 10^{-3} V_a + 2.135 \times 10^{-4} V_a^2 \tag{23b}$$

$$s_x = 0.6 \log_{10}\left[\frac{1.37 \; V_b^2 - 1}{1.33 \; V_b - 1}\right] \tag{23c}$$

$$s_w = 0.76 \; (s_z - s_y) \tag{23d}$$

where V_a is the volume concentration of aggregate (percent)

   V_b is the volume concentration of binder (percent)

These parameters and the binder stiffness, S_b, are used to estimate the mixture stiffness, S_m, in the following relationships:

If $5.0 \times 10^5 < S_b < 10^9$ Pa, then:

$$\log(S_m) = \left[\frac{s_w + s_x}{2}\right](\log(S_b) - 8) + \left[\frac{s_w - s_x}{2}\right]\left|\log(S_b) - 8\right| + s_y \tag{24a}$$

and, if 10^9 Pa $< S_b < 3.0 \times 10^9$ Pa, then:

$$\log(S_m) = s_y + s_w + 2.0\% \; (s_z + s_y + s_w) \; (\log(S_b) - 9) \tag{24b}$$

Thus, for any given mixture, traffic speed and ambient temperature, the stiffness of the mixture, S_b, may be estimated. It will be appreciated that there are five independent variables in this analysis: initial penetration, time of loading, temperature, volume concentration of binder and volume concentration of aggregate. What values of these variables should be used? If the mean values are chosen, then information at the extremes is lost, and such information may be of particular importance. By virtue of using the mean values, the final magnitude of the mixture stiffness will be exceeded fifty percent of the time, - equally the magnitude will be overestimated fifty percent of the time.

Methods exist whereby the distributions of the input variables may be used to provide an estimate of the output distribution. Classically, since the relationships are non-linear, recourse might be had to the First-Order, Second-Moment (FOSM) method[18]. However for the relationships involved in this analysis and the lack of

precision or accuracy relative to several of the variables, the FOSM methodology would be incredibly cumbersome, if indeed it is tractable. Alternatively, the Rosenblueth Point Estimate Method (6,26) was adopted, which is in itself much more flexible in application and more forgiving in terms of the quality of the input information which may be used.

STIFFNESS MODEL INPUT PARAMETERS

As mentioned previously, five input parameters need to be defined in order to complete the stiffness model, these parameters are: initial binder penetration, time of loading, temperature, volume concentration of the binder and the volume concentration of the aggregate. These variables have to be defined for both the reference case, i.e., the AASHO Road Test, and for the conditions prevalent within the State of Indiana.

AASHO Road Test

Initial Penetration

Figure 91 of HRB Special Report 61B(19), gives a cumulative frequency plot of the binder penetration at the Road Test (based on 82 samples taken during the course of construction). The mean penetration and standard deviation from this data are given in Table 3.

Mean Penetration	90.5
Standard Deviation	1.98

Table 3. AASHO Road Test
Initial Penetration

The specified binder was an 85-100 pen asphalt. Based on Table 3, compliance was good, and the material of uniform consistency.

Time of Loading

Traffic at the Road Test was held closely to a speed of 35 mph (56.3 kmph). No information is given in the records of the Road Test about the degree of compliance with this figure. An assumption is made that this stated speed is deterministic and unique. Since the time of loading is approximately equal to the reciprocal of the running speed (kmph), the time of loading at the Road Test may be taken as 0.0178 seconds.

Temperature

Appendix B of the HRB Special Report 61E tabulates the mean maximum and mean minimum air temperatures for each of the 55 two-week periods of the Test. Using this data the air temperature distribution throughout the period of the Test was synthesized, by summing 55 uniform distributions, each bounded by the mean maximum and mean minimum recorded temperatures. The statistics of the resulting distribution are given in Table 4.

Mean Air Temperature ($^\circ$F, $^\circ$C)	50.8	10.44
Standard Deviation ($^\circ$F, $^\circ$C)	20.7	11.48
Skewness (β)	-0.19	

Table 4. AASHO Road Test
Air Temperature Distribution Parameters

In order to make use of the Odemark equivalent stiffness methodology outlined above, the temperature of the pavement is required rather than the air temperature. No information of this kind has been found by the Authors in the published Road Test literature or computer data-base.

In order to synthesize the pavement temperature distribution from the air temperature distribution, recourse was made to a

relationship reported by Witczak(20) which relates the mean monthly
pavement temperature (MMPT) at any depth, z, to the mean monthly air
temperature (MMAT). In this project, predicting the temperature at
any depth at any particular time is of no interest.   It is the
expected (or time averaged) distribution of pavement temperature
that is required. On this basis, it was felt that the level of
accuracy provided by this means would be adequate.

Since Witczak's formula yields the expected temperature at any given
depth t, recourse was made to the mean-value theorem(27) to find the
expected temperature in a layer of z-thickness (i.e., in a layer
starting at the surface (z=0) and extending to a depth of z = t
inches), from which:

$$\overline{T}_p = 6.0 - \frac{34 \, \log_{10}\left(\frac{4}{(z+4)}\right)}{z} + \frac{\overline{T}_a \left(1.0 - \log_{10}\left(\frac{4}{(z+4)}\right)\right)}{z} \qquad (25)$$

where $\overline{T}_p$ is the expected pavement temperature in a surface
          layer z inches thick. ($^\circ$F)

          $\overline{T}_a$ is the expected value of air temperature ($^\circ$F)

The pavement surfacing thickness z, at the Road Test was not
constant. Design thicknesses were 2,3,4,5 and 6 inches depending
upon the section considered (disregarding any variation in
constructed thickness). Since the above relationship indicates that
the mean pavement temperature is a function of the layer thickness,
and the binder stiffness S_b, is a function of the pavement
temperature, then the use of a single value of the layer coefficient
for different thicknesses of the surfacing material may be
questioned.

The AASHO method, however, does indeed assign a single value of
layer coefficient to the surfacing material, regardless of the layer
thickness.   The distribution parameters of surfacing layer
thicknesses at the Road Test are given in Table 5.

Mean Layer Thickness (in)	4.00
Standard Deviation (in)	1.08

Table 5. AASHO Road Test
Layer Thicknesses

Applying Rosenblueth's point estimate method to Equation (25), and using the distribution parameters for air temperature (Table 4) and layer thickness (Table 5), the mean layer temperature and standard deviation are given for the AASHO Road Test pavement surfacing in Table 6.

Mean Layer Temperature (F, C)	51.9	11.05
Standard Deviation (F, C)	24.4	13.58

Table 6. AASHO Road Test
Pavement Temperatures

Volume Concentration of Binder

Evaluation of the binder volume concentration distribution is somewhat complicated by the fact that the bituminous material characterized by the layer coefficient a_1 at the Road Test was not a unique mixture. The Road Test "surfacing" actually comprised two distinct bituminous mixtures laid in combinations of differing thickness. The surfacing mixture (S) was generally laid in a layer 1.5 inches thick, and the binder layer (B) varied in thickness to make up the required total surfacing thickness, as shown in Table 7.

	Surfacing Thickness (in)				
	2	3	4	5	6
Surface Mix	2.0	1.5	1.5	1.5	1.5
Binder Mix	–	1.5	2.5	3.5	4.5
Frequency	1/12	1/4	1/3	1/4	1/12

Table 7. AASHO Road Test
Surfacing/Binder Thicknesses

Since each mixture, Surface (S) and Binder (B) possessed distinctly different mix parameters, the combined effect was different for each total thickness, (another reason why a unique layer coefficient is not reasonable). However, recognizing the variation of thickness and mixture properties allows an explicit treatment through the Rosenblueth Point Estimate Method by developing the required distribution for the combined mixtures and thicknesses.

HRB Special Report 61B reports the volumetric mixture parameters from the density and extraction tests conducted on both bituminous materials. The distribution characteristics are shown in Table 8.

	Surface		Binder	
	Mean	Std.Dev	Mean	Std.Dev
Air Voids (%V)	3.6	0.43	4.8	0.52
Voids Filled (%F)	77.9	2.21	68.2	2.70

Table 8. AASHO Road Test
Mixture Parameters

Denoting the proportion of each surfacing thickness taken up by the surface mix (S) as p, and $(1-p)$ for the corresponding binder mix, then the composite volume percentage of binder (V_b) may be given as follows:

$$V_b = p \frac{(V[S] \times F[S])}{(1 - F[S])} + (1-p) \frac{(V[B] \times F[B])}{(1 - F[B])} \qquad (26)$$

In Equation [26], V[] denotes the volume percentage of air voids, and F[] the volume percentage of voids filled with binder; S and B relate to the surfacing mixture and the binder mixture respectively. Using the Rosenblueth point estimate method yields the distribution parameters given in Table 9 for the combined thicknesses of bituminous surfacing. Note that in this case the skewness β_1 is significant.

Mean V_b (%)	11.32
Std. Dev.	1.58
Skewness β_1	0.58

Table 9. AASHO Road Test
Volume Concentration of Binder (V_b)

Volume Concentration of Aggregate

In a similar way, the combined effects of thickness, air voids and voids filled may be resolved by the Rosenblueth method to give the values in Table 10, where:

$$V_a = p \frac{(1 - F[S] - V[S])}{(1 - F[S])} + (1-p) \frac{(1 - F[B] - V[B])}{(1 - F[B])} \qquad (27)$$

Mean V_a	84.40
Std. Dev.	1.79
Skewness β_1	-0.20

Table 10. AASHO Road Test
Volume Concentration of Aggregate (V_a)

Road Test Variable Summary

All the required parameters for the distributions of the five
significant independent variables needed to derive the distribution
of the AASHO Road Test Surfacing mixture are thus available. These
are summarized in Table 11.

Variable		Mean	Std.Dev	Skewness
Penetration	Pen_l	90.5	1.98	*
Time of Loading (sec)	t	0.0178	*	*
Temperature ($^\circ$C)	T	11.05	13.58	*
Volume Concentration of Binder (%)	V_b	11.32	1.58	0.58
Volume Concentration of Aggregate (%)	V_a	84.40	1.78	-0.20

Table 11. AASHO Road Test
Summary of Variable Statistics

Utilizing these values with the Rosenblueth point estimate method,
the following values for the distribution parameters for the
variables required by the Bonnaure et al. relationships of the
AASHTO Road Test Surfacing were obtained. Since the stiffness S_m in
equations 24 is given as a logarithm, the distribution parameters in
Table 12 are given for the logarithm of S_m, $\log_{10}(S_b)$.

Mean $\log(S_m)$	8.9238
Std. Dev.^{n}	0.6902
Skewness β_1	2.2801

Table 12. AASHO Road Test
Mixture Stiffness (S_m) Statistics

IDOH Input

Initial Penetration

Historically, the State of Indiana has specified 60-70 grade penetration binder. During the 1970's, the asphalt industry changed over to viscosity graded binders, this resulted in the State specifying AC-20 grade binder. While these grades were not the only grades specified, they have both, by far, comprised the greater percentage in actual use.

On any given contract, the specified grade of binder proposed by the Contractor, and accepted by IDOH will, under normal circumstances, be used throughout the Contract. At the same time, the consistency of the binder provided to the Contractor will normally be closely controlled within narrow limits by the manufacturer. A separate Contract in another part of the State might use binder supplied by a different manufacturer, or by the same manufacturer at a different date. Each of the binders supplied by the manufacturer(s) may well comply with the State Specifications, and be well controlled. However, over a period of years the distribution of the consistency parameters (penetration or viscosity) taken overall will be wider than might be expected on any single Contract.

Binders specified under the previous penetration grading system, and the current viscosity grading system have generally complied with the specifications. Fortunately, it is only necessary to analyze the distribution of the binder penetration rather than individual data.

The 1985 IDOH Specification requires that the AC-20 grade binder comply (inter alios) with the consistency criteria given in Table 13.

Viscosity poise	1600 - 2400
Penetration	50 - 110

Table 13. Indiana Specification AC-20 Binder Consistency

The effect of the change in primary consistency variable (penetration → viscosity) is in the wider range of penetration for an acceptable viscosity. In actual practice, the range of measured penetrations is much narrower than that permitted in the Specifications. IDOH no longer routinely test binders for penetration, however McConnaghay Emulsions Inc. still routinely test all binders supplied to them for penetration. They state that the binder in most common use (i.e., AC-20) is still essentially classifiable as a 60-70 penetration binder. However the range is somewhat wider, more nearly 55-80 penetration.

Using this information, the distribution of the initial penetration of binders in current use may be synthesized, and are given in Table 14.

Range	50 < Penetration < 110
Mean	64

Table 14. Indiana Binders
Range and Mean Penetration

Using the beta distribution(26) (for bounded data), the distribution parameters given in Table 15 are developed.

Mean Penetration	64
Standard Deviation	4
Skewness β_1	0.39

Table 15. Indiana Binders
Penetration Distribution Parameters

The resulting distribution complies with all the statements made by IDOH Materials & Testing personnel and McConnaghay personnel with regard to the current distribution of penetration.

Time of Loading

Whereas, at the AASHO Road Test, the traffic speed was tightly controlled, the spectrum of speeds observed on the State Highways and Interstates is much wider. Purdue University maintains an annual Speed-Study program for the State for purposes of Federal Speed Regulation compliance estimation. Consequently, there is a solid basis for estimating the time of loading parameter within Indiana. The data in Table 16 was abstracted from the 1937 Traffic Speed Report(21).

	Mean Speed	Std.Dev	$VMTx10^9$
Urban Interstate	57.99 mph	3.80	3.7565
Rural Interstate	60.99 mph	4.15	4.5488
4 - lane State	57.21 mph	4.70	8.1204
2 - lane State	55.41 mph	5.25	5.5560
All Highways	57.67 mph	4.98	*

Table 16. Speed summary, Indiana Highways 1987
(VMT = Vehicle miles travelled)

The figures given in Table 16 for "All Highways" are the speed parameters for the whole state obtained by weighting the observed values by VMT. Expressing these data in terms of the time of loading which is the parameter of primary interest, the values given in Table 17 are found.

There is really insufficient difference between the results for the individual classes of highways to require them to be treated separately. Consequently, the values given above for "All Highways" will be adopted.

	Mean (sec)	Std.Dev
Urban Interstate	0.0107	0.0007
Rural Interstate	0.0103	0.0007
4 - lane State	0.0109	0.0009
2 - lane State	0.0112	0.0011
All Highways	0.0108	0.0009

Table 17. Indiana
Time of Loading Characteristics

Temperature

The National Oceanographic and Atmospheric Administration (NOAA) maintains climatological data for each state and publishes both daily, monthly and summary statistics for many towns and cities within each state. Detailed summary data for 47 cities and towns in Indiana are available (22) and have been used to provide statistics of air temperature (30yr statistics). These data are summarized in Table 18. On the basis of these statistics, the state of Indiana was sub-divided into two distinct temperature zones, labelled North and South, (Figure 2 shows the temperature contours throughout the state and indicates the dividing line between North and South). Converting the temperatures for the two zones and for the whole state from Fahrenheit to Celsius and using the same method as for the AASHO Road Test to estimate the pavement temperatures, the summary statistics are given in Table 19.

On the basis of these statistics, the state of Indiana was sub-divided into two distinct temperature zones, labeled North and South. The summary statistics are given in Table 19 for both the North and South zones as well as for the state as a whole.
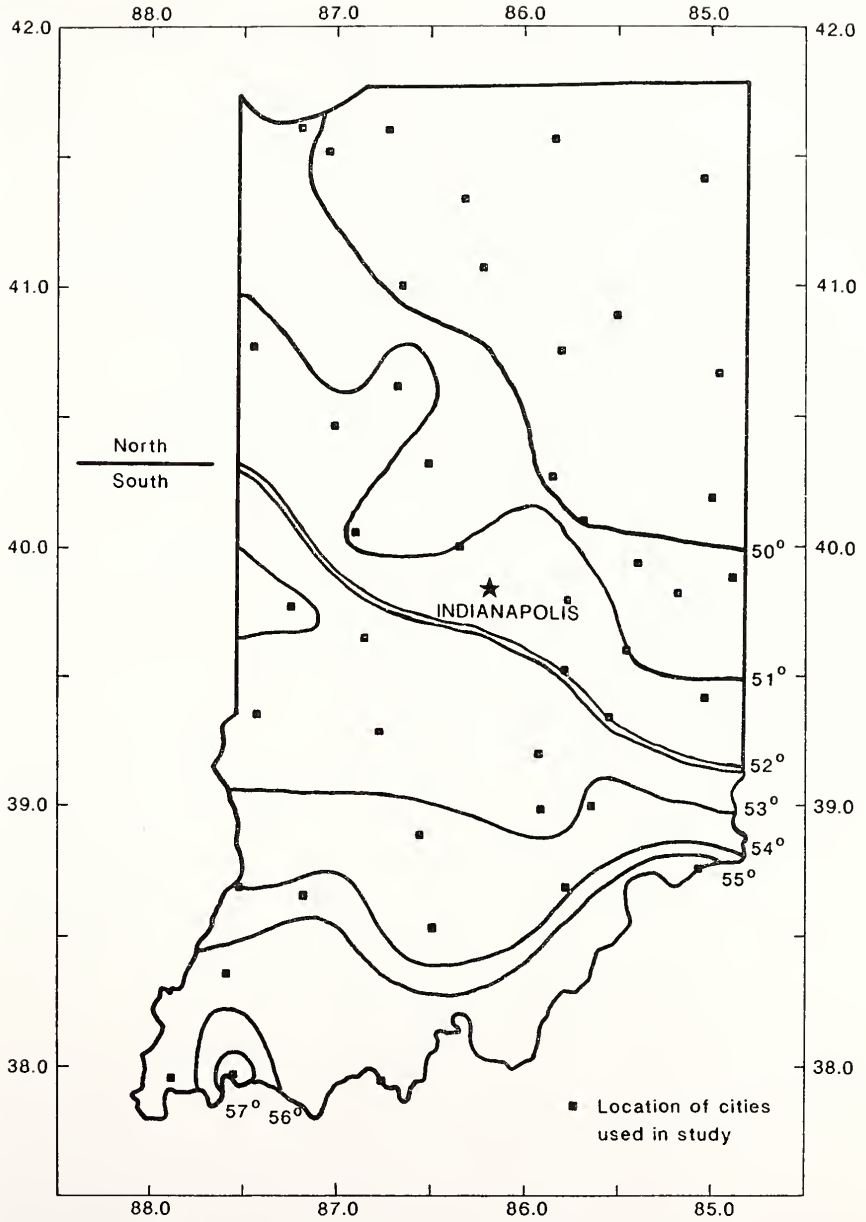
Figure 2. Indiana — Mean Annual Temperature Contours.

City/Town	Zone	Mean °F	Std Dev	Beta β
Anderson	N	51.1792	17.8599	−0.0329
Berne	N	50.9875	18.1846	−0.0126
Brookville	N	51.2458	17.9767	0.0106
Cambridge City	N	50.0292	18.1831	0.0028
Columbia City	N	49.5708	18.4568	−0.0132
Columbus	S	52.4125	17.8518	−0.0168
Crawfordsville	N	50.8458	18.5126	−0.0211
Delphi	N	51.5375	18.2027	−0.0406
Elwood	N	49.8458	18.3799	−0.0221
Evansville	S	57.5208	17.2520	−0.0182
Frankfort	N	50.2958	18.3826	−0.0381
Goshen	N	49.5375	18.2295	−0.0219
Greencastle	S	52.3933	18.5189	−0.0461
Greenfield	N	51.6250	18.3169	−0.0363
Greensburg	N	51.9458	17.4352	−0.0199
Hobart	N	50.6542	18.4627	−0.0423
Huntington	N	50.7000	18.2297	−0.0174
Kentland	N	51.2500	18.8562	−0.0566
LaPorte	N	49.7208	18.5821	−0.0286
Marion	N	49.5500	18.4428	−0.0083
Mount Vernon	S	55.5833	17.7614	−0.0231
New Castle	N	50.1917	18.2610	0.0024
North Vernon	S	54.3458	17.2623	−0.0358
Ogden Dunes	N	50.2958	18.1794	−0.0425
Oolitic	S	53.1250	17.6508	−0.0268
Paoli	S	53.0083	17.7968	−0.0197
Plymouth	N	50.2875	18.7438	−0.0102
Princeton	S	55.4958	17.5530	−0.0331
Richmond	N	50.4875	17.6566	−0.0058
Rochester	N	48.9375	18.7742	−0.0244
Rockville	S	53.0708	18.2021	−0.0579
Rushville	N	50.8792	18.0099	−0.0437
Scottsburg	S	53.7167	17.8155	−0.0116
Seymour	S	52.6625	17.8888	−0.0185
Shelbyville	N	52.0417	18.1413	−0.0442
Spencer	S	52.3125	18.2644	−0.0126
Tell City	S	55.7625	17.3456	−0.0078
Terre Haute	S	52.6042	18.2972	−0.0565
Valparaiso	N	49.5125	18.3281	−0.0655
Vevay	S	54.8917	17.2462	0.0077
Vincennes	S	54.0958	18.2081	−0.0269
Wabash	N	48.9875	18.5948	−0.0191
Washington	S	55.6125	17.3811	−0.0471
Waterloo	N	49.2042	18.4533	−0.0068
West Lafayette	N	50.0375	18.5902	−0.0628
Winamac	N	49.7917	18.4320	−0.0516
Winchester	N	49.7417	18.2160	−0.0390

Table 18. Temperature Statistics for 47 Cities in Indiana

North Zone	N	50.3639	18.3243	-0.0294
South Zone	S	54.0355	17.8495	-0.0347
All	N+S	51.6919	18.2395	-0.0375

Table 19. Temperature ($^\circ$F) Statistics for 47 Cities in Indiana

Converting the temperatures for the two zones and the whole state from Fahrenheit to Celsius and using the same method as for the AASHO Road Test to estimate the pavement temperatures, the values in Table 20 are obtained.

	Mean ($^\circ$C)	Std.Dev
North	11.87	11.96
South	14.24	11.66
All	12.47	12.12

Table 20. Indiana Pavement Temperature Distribution Parameters

The conversion to pavement temperatures relies on the assumption that the range of layer thicknesses used throughout the State of Indiana is similar to that used at the AASHO Road Test, i.e., 2 to 6 inches with a mean of 4 inches.

Volume Concentration of Binder, and Volume Concentration of Aggregate

The 1985 IDOH Specification covers ten mixtures for highway paving from relatively coarse base course mixtures (maximum aggregate size 1+ inches) through sand surfacing mixture (maximum aggregate size 3/8 inch). Each of these mixtures is characterized in the Specification by a gradation envelope and an acceptable range of binder content. The binder content on any given Contract would be tightly controlled, however, the overall variation for a given mixture over a number of years is likely to be quite large. Consequently, the distribution of binder content for each mixture

has been characterized by the uniform distribution between the specified binder content limits.

In the Supplemental Specifications to the 1985 Standard Specifications (1987)(23), the Percent air voids is specified to be in the range 4 to 8 percent for all mixtures. Again, for the same reasons, the air voids have been assumed to be uniformly distributed between the limits.

The binder specific gravity was assumed to be constant at 1.021, and the total aggregate specific gravity was assumed to be a constant 2.65 (aggregates, both individually and in a mixture, vary in terms of their specific gravity, however the error introduced by using 2.65 is assumed to be small when compared to other sources of uncertainty). Table 21 summarizes the volume concentration of aggregate (V_a) and volume concentration of binder (V_b) data for all of the IDOH mixtures.

It may be clearly seen that the two Base mixtures are identical in terms of the parameters of interest. However, in terms of their gradations they are mutually distinct. The implication in this is that while their strengths may be identical, their workabilities might be quite different.

IDOH Variable Summary

The distribution parameters for the five significant independent variables needed, are given in Tables 21 and 22.

MIX	V_a %	$\sigma(V_a)$	V_b %	$\sigma(V_b)$	$\rho(V_a \cdot V_b)$
BASE					
5	83.65	1.228	10.34	0.688	−0.5
5B	83.65	1.228	10.34	0.688	−0.5
BINDER					
8	83.44	1.288	10.56	0.683	−0.5
9	83.02	1.220	10.98	0.679	−0.5
11	82.60	1.214	11.40	0.678	−0.5
SURFACING					
8	81.98	1.273	12.02	0.792	−0.5
9	81.26	1.296	12.74	0.842	−0.5
11	80.14	1.318	13.86	0.887	−0.5
12	79.74	1.310	14.26	0.888	−0.5
SAND	77.66	1.106	16.34	0.598	−0.5

Table 21. Indiana Mixture Parameter Statistics
(σ is the standard deviation; ρ the correlation coefficient)

Variable		Mean	Std.Dev	Skewness
Penetration	Pen_i	64.0	4.00	0.39
Time of Loading (sec)	t	0.0108	0.0009	*
Temperature (°C) North South All	T	11.87 14.24 12.47	11.96 11.66 12.12	* * *

Table 22. Indiana Summary of Variable Statistics

LABORATORY RESULTS

Introduction

Some form of verification of the proposed method is required.   Due
to the deficiencies in the original AASHO method as outlined in
sections 1 and 2, and in reference 28, it is not possible to obtain
a measure of layer coefficients directly, nor is it feasible to run
laboratory tests which duplicate the time-averaging effects inherent
in the layer coefficient concept.

Two aspects of the proposed model lend themselves to verification:
the Odemark equivalent stiffness   method   and   the   van   der
Poel/Ullidtz/Bonnaure et al. relationships. The Odemark equivalent
stiffness method has been adequately verified by others (e.g. (9)),
and has its basis in structural mechanics. On the other hand, the
van der Poel/Ullidtz/Bonnaure et al. relationships are less well
supported. The van der Poel nomograph is well supported, but as
previously mentioned is quoted as having an accuracy within a factor
of two. The Ullidtz relationships reduce the van der Poel nomograph
to equation form within an accuracy of 10 percent. The Bonnaure et
al. relationships rely on the van der Poel nomograph and were
extensively verified by the originating authors, however   the
applicability of these relationships to mixtures other than those
originally tested is not known.

The laboratory phase of this project was specifically undertaken to
provide some verification of the ability of the van der Poel/Ullidtz
/Bonnaure et al. relationships to adequately predict the modulus, or
stiffness S_m, of mixtures in Indiana.

Materials Tested

Six samples were prepared for testing in the Laboratory. One sample was made up in the laboratory to comply with the 1985 IDOH Standard Specification for sand mix surfacing. The other samples were cut from core samples taken in the course of a recent JHRP research project(25). One of these (F-15) was cut to obtain a sample of fine surfacing mixture, however since no single surfacing thickness could be found of adequate thickness for testing (3 inches or greater) this sample contained two distinct surfacing layers (approx. 3/4 inch and 2 1/4 inch cut). The remaining four samples were selected to represent coarser base mixtures. The basis of selection for the base mixtures was to select visually comparable paired samples such that one pair represented similar (crushed aggregate) mixtures from both North and South zones, and similarly the other pair contained rounded aggregate.

In this way, it was hoped that the gradation range (fine to coarse) and the material range (rounded and angular) would be represented in the testing program. At the same time, the contractor bias could be reduced by selecting samples from different areas of the State.

Test Method

The Uniaxial Static Creep test recommended by Bolk et al. (24) was selected to measure mixture stiffness. Other forms of creep testing are available, but the majority rely on repeated cyclical loading over two or more minutes. These latter are designed to simulate the mechanism of rutting, while the uniaxial static creep test is more relevant in the case of single load applications and provides information in the time interval of typical traffic wheel passage.

Samples for the creep test were cut from the selected cores using a diamond rock saw. The ends were checked for parallelism, and any corrections made with a thin layer of Plaster of Paris. The samples were conditioned for at least 24 hours to uniformly attain the required test temperatures (5°C, 18°C and 39°C); a dummy sample with

an internal thermometer was held in the conditioning chamber as a reference to control the sample temperatures.

The samples were placed between the platens of the MTS apparatus. A double layer of heavy-duty polythene sheeting was placed between both ends of the sample and the platens to reduce end friction effects. In addition the interface between the polythene sheets was coated with silicone grease. A load of 265 lbs was applied as a step function to the sample with the MTS machine, and load and deformation measurements from the MTS 500 lb load cell and vertical LVDT were recorded by the automated data acquisition system (LAB-TECH Inc. on an IBM PC-XT) at intervals of 1/1000 seconds.

After testing, the samples were returned to the conditioning chamber to equilibrate to the next temperature. The sequence of temperatures $(5^{\circ}C \rightarrow 18^{\circ}C \rightarrow 30^{\circ}C)$ was chosen to minimize the effect of sample disturbance.

The stiffness S_m of the mixtures was computed at load durations of 1/100 second and 1/10 second, to emulate the effects of vehicular loading at approximately 60 mph and 6 mph. The sample lengths L, and diameters (for cross-sectional areas) were measured by micrometer to 0.1 mm, five measurements of each provided mean values). The stiffness was computed as:

$$S_m = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Load/Area}}{\text{Deformation/Sample Length}} \tag{28a}$$

$$S_m = \frac{P/A}{\delta(t,T)/L} = \frac{P \cdot L}{A \cdot \delta(t,T)} \tag{28b}$$

The designation $\delta(t,T)$ denotes the deformation under a load duration of t seconds at a temperature of $T^{\circ}C$. Results of the stiffness calculations are given in Table 23. An idealized plot of the data is given in Figure 3, showing the relationship between load, displacement and time.
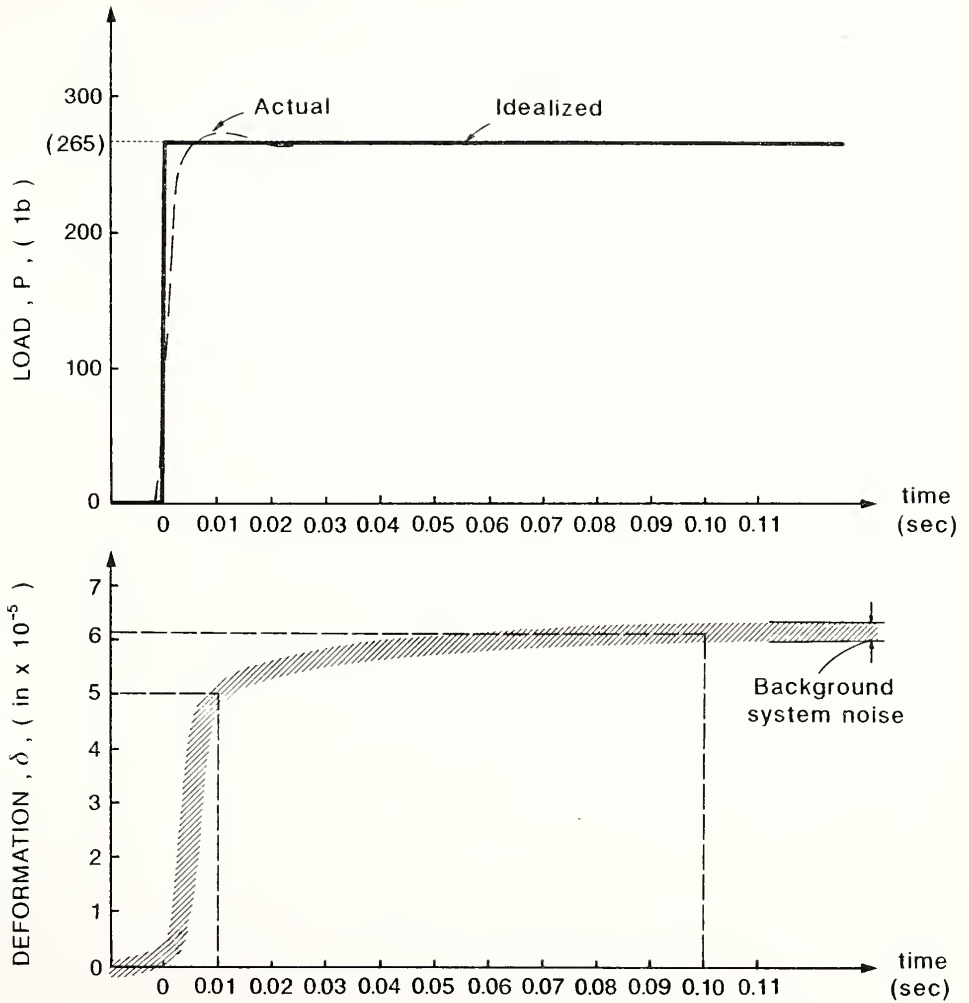
Figure 3. Load-Deformation Test Schematic.

Sieve	Sample					
	Sand	F-15	V-9	F-18	L-12	S-12
1 1/2	100	100	100	100	100	100
3/4	100	100	77	77	80	82
3/8	100	78	45	41	47	52
# 4	96	57	33	27	27	33
# 8	77	42	28	21	20	27
# 16	49	31	21	17	16	20
# 30	33	22	14	14	13	13
# 50	19	11	6	8	7	5
# 100	12	7	2	3	3	2
# 200	5	4	1	1	1	1
% Bind	7.4	5.3	4.9	4.7	4.8	5.1
MIX	Sand	???	5	5	5	5/5B

Table 23. Materials. Summary of Extraction Tests

Bulk density (AASHTO T166) was obtained by weighing the samples dry
and in water. Specific gravity was determined by Rice's method
(AASHTO T209). The binder content was determined by centrifuge
extraction (AASHTO T164, Method A) with trichloroethylene and the
washed gradation (AASHTO T30) found. This information permitted the
calculation of the parameters needed for the Bonnaure et al.
relationships (V_a and V_b), and allowed an estimate to be made of the
original mixture designations which are shown in Table 23.

Mix	Temp °C	0.01 seconds		0.10 seconds	
		Measured	Computed	Measured	Computed
SAND	5	1307000	1528000	817000	951000
	18	545000	609000	284000	321000
	30	133000	148000	70000	79000
F-15	5	2261000	2140000	1508000	1440000
	18	1131000	969000	646000	547000
	30	283000	274000	162000	156000
V-9	5	2146000	2607000	2146000	1831000
	18	1073000	1248000	715000	711000
	30	286000	361000	159000	206000
F-18	5	2196000	2666000	2196000	1883000
	18	1464000	1293000	732000	745000
	30	488000	383000	274000	221000
L-12	5	2176000	2451000	1451000	1699000
	18	1088000	1160000	725000	664000
	30	363000	339000	207000	194000
S-13	5	2173000	2707000	1448000	1916000
	18	1086000	1299000	621000	733000
	30	272000	367000	150000	207000

Table 3. Comparison of Mixture Stiffnesses (S_m)
Measured vs Computed (psi)

Results

The results of gradation and extraction tests are given in Table 23, as are estimates of the mixture designations. Table gives the res the laboratory stiffness tests and the projected stiffness from the estimation procedure. A number of preliminary obset may be made. There is little difference in r the measur computed stiffnesses between the mixtures rounded ags (F-18, L-12) and those with crushed ates (V-9, The overall agreement between the measured outed stif is in general quite good (see Figure 4);
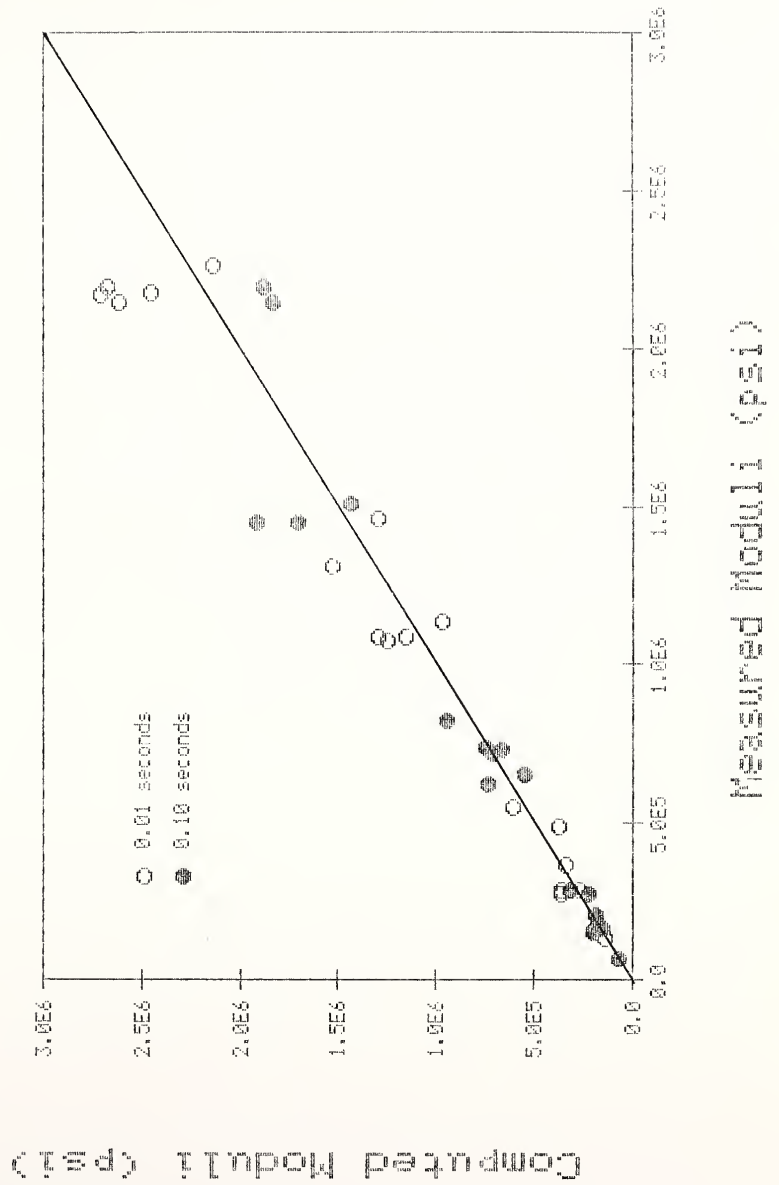
Figure 4. Computed vs. Measured Moduli.

using as a measure of agreement the ratio "measured/computed", the mean value is 0.96 and the standard deviation 0.152 which gives a probable range (mean plus/minus 3 standard deviations) of 0.50 to 1.41; this agrees well with the van der Poel and Bonnaure et al. estimates of the accuracy of the component parts of the model.

It should be noted that at the $5^\circ C$ test temperature, the recorded deformations were of the order of 0.00002 inch, while the MTS equipment is capable only of reading to an accuracy of 0.00001 inch. Thus, a reading of 0.00002 inch might realistically represent any value between 0.000015 and 0.000025 inches, the corresponding possible error in Equation 28 is significant. The percentage error of the same type at the $18^\circ C$ and $30^\circ C$ temperatures is correspondingly smaller.

The laboratory test results provide reasonable verification for the van der Poel/Ullidtz/Bonnaure et al. relationships. There is certainly little evidence that the relationships are invalid.

SUMMARY

The distribution parameters for the AASHO Road Test and IDOH mixtures have been applied to the van der Poel/Bonnaure et al. relationships. As a result, three possible approaches to estimating layer coefficients for IDOH bituminous mixtures may be considered. These approaches address deterministic, a combination of deterministic and probabilistic, and probabilistic concepts.

i.   Deterministic. (i.e., only using the mean values of each parameter or variable). In this approach, the expected values of each of the final distributions were used alone, such that:

$$a_{idoh} = E\left(a_{aasho}\right) \cdot \left[\frac{E\left(S_{m_{idoh}}\right)}{E\left(S_{m_{aasho}}\right)}\right]^{1/3} \qquad (29)$$

ii. Mixed Deterministic/Probabilistic. The value used for the AASHO layer coefficient a_{aasho} is treated deterministically and assigned its mean value. The two stiffness variables, $S_{m_{idoh}}$ and $S_{m_{aasho}}$ are treated as distributions, such that:

$$a_{idoh} = E\left(a_{aasho}\right) \cdot \left[\frac{D\left(S_{m_{idoh}}\right)}{D\left(S_{m_{aasho}}\right)}\right]^{1/3} \qquad ** \qquad (30)$$

iii. Probabilistic. In this approach, all three final variables, $a_{m_{aasho}}$, $S_{m_{aasho}}$ and $S_{m_{idoh}}$, were treated as distributions, such that:

$$a_{idoh} = D\left[a_{aasho}\right] \cdot \left[\frac{D\left[S_{m_{idoh}}\right]}{D\left[S_{m_{aasho}}\right]}\right]^{1/3} \qquad ** \qquad (31)$$

**   D denotes that the distributional parameters (μ, σ, β) of these variables have been used.

NORTH						
MIX	Deterministic	Mixed		Probabilistic		
	Mean	Mean	σ	Mean	σ	
BASE						
5/5B	0.466	0.483	0.114	0.552	0.299	
BINDER						
8B	0.464	0.492	0.100	0.554	0.299	
9B	0.460	0.489	0.101	0.552	0.297	
11B	0.456	0.486	0.101	0.549	0.294	
SURFACING						
8S	0.451	0.482	0.102	0.544	0.293	
9S	0.444	0.477	0.102	0.538	0.290	
11S	0.435	0.470	0.103	0.530	0.288	
12S	0.431	0.468	0.105	0.527	0.284	
Sand	0.414	0.455	0.104	0.512	0.277	

Table 25. Indiana Layer Coefficient Summary (North)

As may be seen from the values tabulated in Tables 25 and 26, the computed mean values of layer coefficient for each type (deterministic, deterministic/probabilistic and probabilistic) are distinct, and in fact differ from each other by an almost constant amount. For the mixed deterministic/probabilistic model and the probabilistic model, the standard deviations are essentially constant in each, 0.1 and 0.28 respectively. There is not a large variation between the coarsest mixture (5&5B) and the finest mixture (SAND) in each climatic zone, 0.466 to 0.414 in the Northern zone and 0.449 to 0.396 in the Southern; and yet with reference to the

AASHO mean (0.427), these represent a range of 109 percent to 97 percent in the North and 105 percent to 93 percent in the South.

A word of caution should be sounded here in relation to the results of the probabilistic approach, since this relies on accepting the distribution of the AASHO layer coefficient a$_1$ as being true. It is believed that while the distribution shown in Figure 1 is correct for the AASHO model as originally derived, it is not a true representation of the layer coefficient in the pavement.

SOUTH					
MIX	Deterministic	Mixed		Probabilistic	
	Mean	Mean	σ	Mean	σ
BASE					
5/5B	0.449	0.477	0.098	0.539	0.289
BINDER					
8B	0.447	0.475	0.098	0.526	0.312
9B	0.443	0.472	0.098	0.533	0.286
11B	0.439	0.469	0.098	0.530	0.285
SURFACING					
8S	0.434	0.464	0.099	0.524	0.282
9S	0.426	0.460	0.099	0.519	0.279
11S	0.417	0.452	0.100	0.509	0.274
12S	0.414	0.450	0.100	0.506	0.273
Sand	0.396	0.436	0.101	0.490	0.266

Table 26. Indiana Layer Coefficient Summary (South)

Figure 5 shows the distributions of the layer coefficients for (i) the AASHO Road Test, (ii) the 5&5B Base mixture in the Northern zone, and (iii) the SAND mixture in the Southern zone. These latter were chosen to represent the "stiffest" and the "weakest" of the Indiana mixtures. It will be seen that:

50
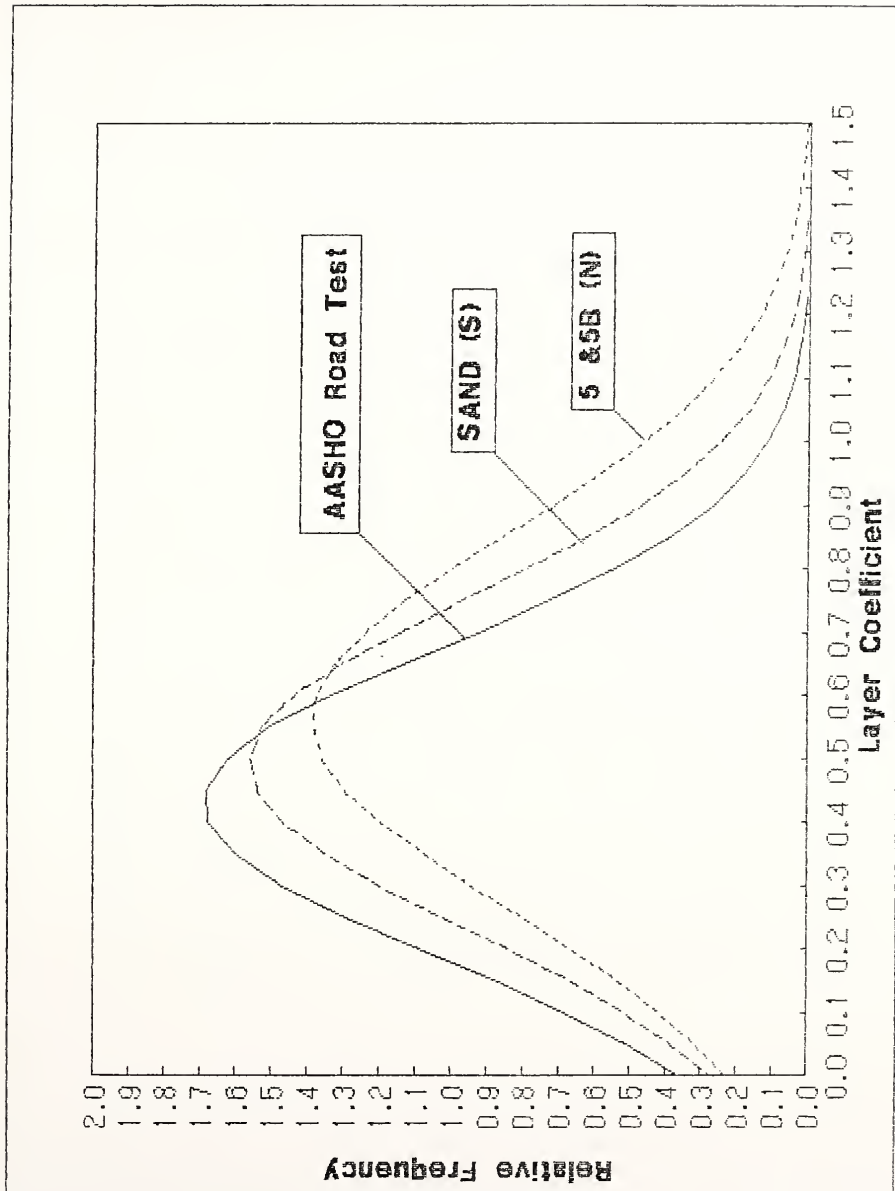
Figure 5. Comparison of AASHO Road Test and IDOH Layer Coefficient
           Distributions

i.   the 'envelope' of IDOH mixtures (represented by the two
      extreme distributions mentioned above) is relatively narrow.
      and

ii.   that this envelope is considerably higher than the AASHO Road
      Test distribution, i.e., 'stronger'.

Generally these may be explained thus:

i.   The mixtures represented in the IDOH specification represent a
      relatively narrow sub-set of asphaltic concretes. Truly coarse
      (open-graded) mixtures and truly fine mixtures (tending to
      mastic) are not represented.   The two most significant
      variables within this envelope are the binder type, and the
      temperature regime. In Indiana the binder type is essentially
      homogeneous (60-70 pen or AC-20) and the temperature
      distribution is not dissimilar from that at the AASHO Road
      Test.

ii.   Given that the temperature distributions at the Road Test and
      in Indiana are similar, then the major discriminant variable is
      the binder type.   At the Road Test, the binder was
      characterized as 85-100 pen, while in Indiana the predominant
      binder (both historically and currently) may be taken as 60-70
      pen. Thus the Indiana binders are 'stiffer' than that used at
      the Road Test. This explains the major shift in the layer
      coefficients which is not compensated by the generally slightly
      warmer temperatures in Indiana.

      The apparent shift between the deterministic and probabilistic
      sets of values may be explained by the assymetrical $(\beta_1 \neq 0)$
      distributions involved.   Had the distributions of a   ,
      $S_{m_{idoh}}$ and $S_{m_{aasho}}$ been symmetrical, then the tabulated values
      of a_{idoh} would have been identical regardless of the model,
      only the "spread" would have varied.

RECOMMENDATIONS

The values tabulated in previous section represent the layer coefficients for the ten IDOH specified mixtures as calculated by the van der Poel, Bonnaure et al., Rosenblueth method and approximately corroborated by Laboratory testing on real mixtures, (one laboratory prepared sample, and five core samples).

The absolute magnitudes selected for use by IDOH based on these results are a matter of in-house policy decision, however, the relative magnitudes are considered to represent in-service reality.

It is recommended that the layer coefficients for the base and binder mixtures be combined and the surfacing mixture layer coefficients by divided as shown below:

Mix	North					South				
	i	ii		iii		i	ii		iii	
	Mean	Mean	σ	Mean	σ	Mean	Mean	σ	Mean	σ
Base/Binder	0.46	0.49	0.10	0.55	0.30	0.44	0.47	0.10	0.53	0.29
Surface 8 & 9	0.45	0.48	0.10	0.54	0.29	0.43	0.46	0.10	0.52	0.28
Surface 11 & 12	0.43	0.47	0.10	0.53	0.28	0.42	0.45	0.10	0.51	0.27
Surface Sand	0.41	0.46	0.10	0.51	0.28	0.40	0.44	0.10	0.49	0.27

Table 27. Indiana Recommended Layer Coefficients

(NB. i, ii, iii, refer to Deterministic, Mixed and Probabolistic)

Any one column (necessarily the same North and South) of layer coefficients may be used as long as it is consistently used. The

mixed type of layer coefficient (column (ii) above) represents the
'safest' compromise between the uncertainty of the original AASHO
layer coefficient distribution (Figure 1), and the simplistic
deterministic layer coefficients (column (i) above).

The use of the word *safest* above, is relative, and relates to the
uncertainty of information. A deterministic value may be considered
as having no uncertainty, while information contained in a
distribution is inherently uncertain. The 'flatter' a distribution
(i.e., the larger the variance with respect to the mean) the greater
the uncertainty. The *true* value of the variable may lie *anywhere*
within the limits of the distribution, but its exact value cannot be
known. Taking the #8 surfacing mixture from the Southern zone, it
will be seen that the probable range of the layer coefficient (mean
± 3 standard deviations) is 0.167 to 0.761 for the mixed type, and
-0.322 to 1.370 for the probabilistic type. In this case, *even if
an assumed value is wrong*, the absolute error in the mixed case will
be less than in the probabilistic case.

Using the mixed type of layer coefficient is also the more
conservative option, since the magnitude is less than the
probabilistic value. In this case a marginally thicker layer will
be required by design, thereby providing a measure of safety margin.

The AASHO method does not explicitly permit the use of other than a
single coefficient to represent the asphaltic component in a
pavement, however, considering the linear relationship between
Structural Number (SN) and the layer thicknesses, $SN = a_1 \cdot t_1 + a_2 \cdot t_2
+ a_3 \cdot t_3$ there is no valid reason why this may not be transformed
thus:

$$SN = (a_{11} \cdot t_{11} + a_{12} \cdot t_{12} + \cdots) + a_2 \cdot t_2 + a_3 \cdot t_3 \qquad (32)$$

where the subscript ij relates the jth sub-layer of the ith "AASHO"
layer.

Regardless of which column of Table 27 might be adopted, it is
considered vital that the overall pavement thickness requirements as

outlined in the Reference 28 be taken into account. The authors feel
that this requirement overrides the need to optimize the pavement
structure within the total pavement thickness; at least insofar   as
the total pavement thickness has been shown to govern the   life
capacity of the pavement, and given an adequate total   pavement
thickness, the thickness design of the individual layers governs the
level of maintenance required throughout that life.

FURTHER RESEARCH

While the Authors believe that the objectives of this research project have been adequately addressed, many questions have been raised, and potential applications of the resulting methodology perceived.

Crushed vs Uncrushed Aggregates

The method used did not differentiate between the performance of crushed vs uncrushed aggregates. The form of the creep test used was such that the duration of load was very short (0.01 to 0.10 seconds). It is likely that a *longer* load duration is needed to discriminate between responses of the different aggregate particle shapes.

The effect of particle shape in a bituminous mixture is further influenced by the overall gradation (gravel vs sand), and may be greatly influenced by the particle shape of the sand particles. Particle shape and mechanical soundness are of greater importance in open-graded mixtures and bituminous macadams, than in continuously graded dense asphaltic concretes.

The effect of particle shape in a bituminous mixture should be studied, *but only in a full study of such factors as stone/sand ratios, aggregate/binder ratios and filler/binder ratios.* Since all of these factors are closely related with each other, and with the effects of voidage, it is very important that all of these be studied together in a coherently planned project.

Mixtures Studied

In the current study, the range of mixtures studied comprised a small subset of the full range of competent bituminous mixtures. Since the study provides a means whereby the strength characteristics of mixtures may be estimated based upon the mixture characteristics, *existing* mixtures may be examined in terms of sensitivity to variations in binder content, air voids, VMA and voids filled. Equally, *new or proposed* mixtures may be examined to predict potential behavior.

The van der Poel/Ullidtz/Bonnaure et al. relationships are based upon unmodified residual bituminous binders. Increasing use of rubber and polymer-modified binders may sufficiently alter the behavior of mixtures such that these relationships no longer hold. The power of this method is such that an effort to extend their general validity to modified binders would be well justified.

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APPENDIX A

THE AASHO FLEXIBLE PAVEMENT DESIGN METHOD:
FACT or FICTION?

THE AASHO FLEXIBLE PAVEMENT DESIGN METHOD:

FACT or FICTION?

[This Appendix comprises the text of a paper presented to the 72nd
Annual Transportation Research Board Meeting, Washington DC, January
1982]

1. INTRODUCTION

The American Association of State Highway and Transportation Officials,
(AASHTO), flexible pavement design method is now well established:
indeed the final edition of a series of Interim Guides was recently
(1986) adopted by AASHTO. This document (1) adds significant volume and
some additional factors to the design process. Specifically, the new
AASHTO Guide incorporates the original development of the Road Test Data
with more recent additions relating to sub-surface (internal) drainage,
materials, reliability and others.

One interesting observation is that the developments in the new Guide
appear to accept the original AASHTO formulations as a starting point.
Taking this approach ignores the wealth of information contained in the
original AASHO Road Test data. New analytical techniques, additional
years of experience with pavement performance, and much hard thinking on
the results of the Road Test provide an opportunity that was missed in
developing the new Guide. In lieu of a fundamental approach, the
developments presented in the new Guide that affect the design procedure
were evolved by a combination of theoretical and empirical models. The
cause and effect of the actual AASHO Road Test performance is not
incorporated.

The AASHTO methodology is used by a number of states in some form or another. In many cases, states have modified details of the method to better suit their peculiar circumstances and experience. Other states have made modification only to the various constants in the original formulation. Overall, some of the statistical constants and coefficients originally developed in the analysis of the Road Test results have taken on physical meanings that are not valid.

A recent research project initiated by the Indiana Department of Highways (IDOH) required the Authors to determine the "layer coefficients" for the ten bituminous mixtures currently being specified in Indiana. IDOH uses the original AASHO formulation with their own set of layer coefficients. However, IDOH has recognized that while the range of mixtures specified represents a vast array of performance characteristics, the assignment of a single layer coefficient to characterize them is both unrealistic in engineering terms and inefficient in financial benefit.

As part of the preliminary research task, a literature review was undertaken. In the review of the literature, a specific effort was made to establish the definition of layer coefficient and the original method for calculating this parameter. The results of this otherwise simple task initially proved to be highly disturbing. Subsequently, further analysis highlighted some very interesting possibilities. This paper deals with the findings and conclusions of this preliminary part of the project.

Much of this preliminary research is definitive, while some is still speculative, since the project is on-going.

2. LAYER COEFFICIENT

Considerable disagreement is apparent as to both the definition and the recommended method of measurement of layer coefficients.

In the 1986 AASHTO Guide(1), the following statements are quoted:

1.2..."*The structural number is an abstract number.... converted to actual thickness of surfacing, base and subbase, by means of appropriate layer coefficients representing the relative strength of the construction materials*"

and

"*In effect, the layer coefficients are based on the elastic moduli M_R and have been determined based on stress and strain calculations in a multilayered pavement system*"

and

2.3.3...".. *it is not essential that elastic moduli of these materials are characterized. In general, layer coefficients derived from test roads or satellite sections are preferred.*"

At the International Conference on the Structural Design of Asphalt Pavements (2), Shook and Finn stated that:

"*It is believed that the coefficients a_1, a_2, a_3 are functions of the strengths of the various layers involved. At the present time (1962), however, no entirely satisfactory techniques are available for defining or measuring these strength factors*".

Perusal of existing and current literature reveals two predominant methods have been adopted for estimating the layer coefficients of bituminous materials: (a) A power law relating the layer coefficient to the Resilient Modulus, M_R [e.g. (1), Fig. 2.5], and (b) based on Odemark's equivalent stiffness hypothesis (3), an analogous relationship is used, wherein the one-third power of the ratio of the material modulus to that of a reference material (whose layer coefficient is presumed known) gives the ratio of the unknown layer coefficient to that of the reference material.

Assuming a relationship between strength and the layer coefficient is a surprising extrapolation, since no measure of structural strength or

adequacy was included in the data used to calibrate the AASHO model. The only variables used in the AASHO model (4) are given in Table 1.

It is readily apparent that none of these variables are measures of strength, although it is conceded that the layer thicknesses may be indirect strength indicators. Further, no cognizance was given to the annual cyclic subgrade strength. Instead, the subgrade strength was assumed to be uniform throughout the Road Test, as indeed was climate.

Table 1. AASHO Road Test Measured Variables

FACTOR	VARIABLE DESCRIPTION	VARIABLE	UNITS
TRAFFIC	i) Number of Axle Repetitions ii) Axle Weight iii) Axle Type	W L_1 L_2	# kips 1 = single 2 = tandem
PAVEMENT	iv) Surfacing Thickness v) Base Thickness vi) Subbase Thickness	t_1 t_2 t_3	in. in. in.
DISTRESS	vii) Extent of Cracking ix) Extent of Patching x) Slope Variance (Roughness) xi) Rut Depth	C P SV RD	$ft^2/1000\ ft^2$ $ft^2/1000\ ft^2$ in.

The AASHO statistical flexible pavement design model was set up to integrate the effects of (i) traffic, (ii) pavement (materials and layer thicknesses) and (iii) serviceability (or level of distress). These effects were measured either directly or indirectly through the factors in Table 1. The development of the AASHO model is given below. A review of this development shows that the layer coefficient is a *secondary* parameter, whose significance is as a regression coefficient, or a calibration constant. If any physical significance is attributed to the layer coefficient, the significance should be relative to the *resistance to distress*, rather than to the structural capacity of the layer material.

3. AASHO MODEL

3.1 Background

The basis of the AASHO model is a decay curve, wherein it is assumed
that the condition of a pavement will deteriorate, or decay, with
accumulated traffic (this implies a certain element of time). In order
to implement this model, the concept of functional pavement
serviceability was developed. This resulted in the composite measure
known as the present serviceability index, (PSI),which results from a
regression equation relating the distress measurements given in Table 1
to an aggregated subjective rating of the adequacy of the pavement by a
panel of adjudicators. A scale of 0 to 5 was assigned to the panel
rating and subsequent qualitative serviceability index. A serviceability
of 5 is the ideal, or perfect, pavement. Pavement serviceability has
several important characteristics:

Initial Serviceability, p_o, is considered to be the
serviceability of the freshly constructed, untrafficked pavement. The
ideal pavement has to be rare. In fact, newly constructed flexible
pavements at the Road Test reflected an average serviceability index
value of 4.2.

Terminal Serviceability, p_t, is considered to be that level of
serviceability at which the pavement is deemed to be no longer
performing its required function. The lower, limiting value of p_t at the
Road Test was 1.5.

Present Serviceability, p, is the measured, or estimated,
level of serviceability at any time during the life of the pavement.
Under normal circumstances then, $p_o > p > p_t$.

Using these definitions, the AASHO model may be stated:

$$\left(\frac{W}{\rho} \right)^{\beta} = \frac{p_o - p}{p_o - p_t} \qquad (1)$$

where W is the number of axle (18 kip) repetitions which will reduce the

serviceability from p_o to p, ρ represents the number of axle repetitions at terminal serviceability, p_t, and β is a shape factor. Simply stated the cumulative traffic at any time as a proportion of the traffic capacity of the pavement is represented as a power function of the proportion of usable serviceability consumed.

By a simple mathematical re-arrangement, the more familiar form of the AASHO relationship is obtained:

$$\log_{10}(W) = \log_{10}(\rho) + \frac{\log_{10}\left[\frac{p_o - p}{p_o - p_t}\right]}{\beta} = \log_{10}(\rho) + \frac{G}{\beta} \qquad (2)$$

where G represents the logarithm of the serviceability ratio.

3.2 Calibration

On each section of the Road Test, the present serviceability p, and traffic W, were determined at intervals of two-weeks throughout the life of the section. The unknown parameters, β and ρ, were obtained by regression analysis. The summarized data used for the original regression analysis is in the form shown in Table 2. In Table 2, the logarithm of traffic is given for serviceabilities of 3.5, 3.0, 2.5, 2.0 and 1.5. These five (5) points then provided the data for the regression used to obtain estimates of β and ρ. It should be noted that on a log-log plot of W vs. G the relationship in equation (2) is linear.

Having obtained the two parameters, β and ρ, for each section, it was assumed that these parameters were functions of the section design (i.e., thickness) and traffic type. On this basis the following functional relationships were assigned to β and ρ:

$$\rho = A_o \cdot (D + 1)^{A_1} \cdot (L_1 + L_2)^{A_2} \cdot L_2 \qquad (3a)$$

$$\beta = 0.4 + B_o \cdot (D + 1)^{B_1} \cdot (L_1 + L_2)^{B_2} \cdot L_2 \qquad (3b)$$

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FLEXIBLE PAVEMENT, UNWEIGHTED APPLICATIONS
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Table 2. AASHO Road Test Serviceability Data

In these equations L_1, L_2 and D (where D is the total pavement thickness) were known for each section. The eight unknown constants A_{0-3} and B_{0-3} were obtained by regression analysis. A variant regression was conducted in which the thickness index D, was given by $D = a_1 \cdot t_1 + a_2 \cdot t_2 + a_3 \cdot t_3$, such that "... *these coefficients were permitted to vary so that the three elements of the pavement structure might each enter into the thickness index, D, with a different weight per unit thickness*", (3). This linear combination of the layer thicknesses provided a better regression and was retained in the model: the transformed thickness (thickness index, D) becoming better known as the Structural Number, SN, and the coefficients a_{1-3} as the layer coefficients. The very well known values for the layer coefficients obtained from the Road Test are given in Table 3.

Table 3. AASHO Road Test Layer Coefficients

	a_1	a_2	a_3
Weighted Traffic	0.44	0.14	0.11
Unweighted Traffic	0.37	0.14	0.10

The materials to which these coefficients relate are : a_1 — Asphaltic concrete, a_2 — crushed limestone base, and a_3 — sand-gravel base. As listed, the raw traffic data was referred to as "unweighted", while the "weighted" traffic was adjusted in an empirical fashion to attempt to take into account the varying effect of the annual climatic cycle upon the subgrade. This technique was the only concession originally given to the effect of climate or variable subgrade support.

3.3 Interpretation

The review of the AASHO Road Test analysis showed the "layer coefficient" to be no more than a regression coefficient with no truly ascribable physical or engineering meaning other than being a form of scaling or normalizing constant. Certainly, no connotation of strength could be assumed since no measurement of strength had been used in its derivation. As a result of this situation, some concern developed about

being able to discriminate the strength and performance characteristics of the asphalt mixtures being specified by IDOH. At this stage the Authors deemed it wise to determine the variability (distribution) of the layer coefficients found at the Road Test. The distributions were obtained by using the original equations (3a and 3b), but solving for the layer coefficients section-by-section rather than globally. Thus the layer coefficients, a_{1-3}, for each section were determined. Since the materials in each layer were designed to be uniform throughout the Road Test, variation in the value of the layer coefficients was expected to be a minimum.

The resulting distributions are shown in Figure 1. The only adjustment made to these distributions was to constrain the layer coefficients to be non-negative. Such a constraint may be explained on the basis that regardless of whether strength or serviceability was to be considered, an increasing layer thickness must lead to an increase in the overall strength or serviceability. Consideration of the layer coefficient distributions in Figure 1 caused considerable concern. The degree of variation, particularly in the surfacing (the most closely controlled material), was surprising. In a specific review of the literature, a measure of confirmation was provided by the values given in Table 10 of HRB SR 61E, in which it was shown that the value of a_1 varied from 0.33 to 0.83 (weighted), and from 0.33 to 0.78 (unweighted).

Considering the degree of control exercised at the Road Test, further research and analysis was demanded to resolve the apparently *fatal* layer coefficient variability represented in Figure 1. The term *fatal* is emphasized because discriminating the structural quality of different asphalt mixtures when the single asphalt mixture at the Road Test

resulted in such great variability seems futile. Consequently, after checking both the methodology and the computations which were found to be substantially correct, a copy of the original Road Test data for the flexible pavement sections was obtained from the Transportation Research Board. This detailed data-base provided qualitative improvement in the data being analyzed.
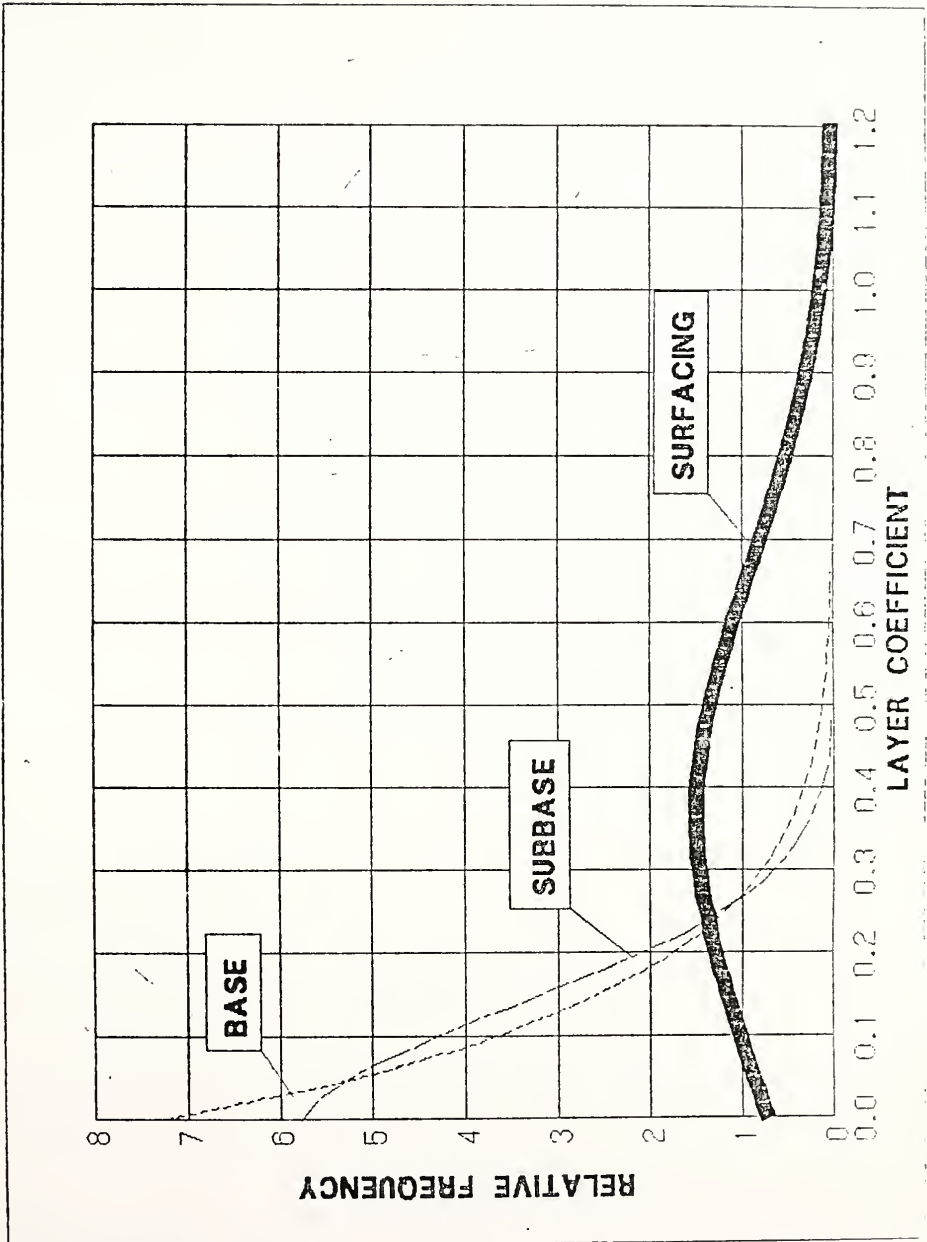
Figure 1. AASHO Road Test – Layer Coefficient Distributions

3.4 Remarks

In the remainder of this paper, the raw, or unweighted traffic data will be used, since the weighting function is somewhat arbitrary and would be difficult to transform for application to sites or climes other than Ottawa. Illinois.

For refernce, the final relationships used by AASHO after the final regressions are given for the standard 18-kip single axle (standard axle) configuration:

Weighted:

$$\log_{10}(W) = 9.61 \cdot \log_{10}(SN + 1) - 0.2 + \frac{\log_{10}\left[\frac{4.2 - p}{2.7}\right]}{0.4 + \frac{1096}{(SN + 1)^{5.19}}} \qquad (4a)$$

Unweighted:

$$\log_{10}(W) = 8.94 \cdot \log_{10}(SN + 1) + 0.35 + \frac{\log_{10}\left[\frac{4.2 - p}{2.7}\right]}{0.4 + \frac{140155}{(SN + 1)^{8.73}}} \qquad (4b)$$

4. ROAD TEST DATA

The remaining Road Test data held by the Transportation Research Board is a reduced set of the original data. Many files, folders and card decks which had not been used have been deleted, the remainder, transferred to computer tape have been preserved. Of particular value, the results from the full factorial experiment have been retained.

The data from the full factorial experiment were reduced to permit easier manipulation, the data fields given in Table 4 were maintained.

Table 4. Retained AASHO Road Test data format

Field #	Description	Field #	Description
1	Loop No.	7,8	Extent of Cracking
2	Lane No.	9,10	Extent of Patching
3	Section No.	11,12	Slope Variance
4	Surfacing Thickness	13,14	Rut Depth
5	Base Thickness	15	PSI
6	Subbase Thickness	16	Accumulated Traffic
		17	AASHO Day

In this file, complete observational data is available for each section for each AASHO day (1 AASHO day = 14 real days). Not only can the PSI (present serviceability index) vs. Traffic history for each section be reconstructed, but the component parts of the PSI (cracking, patching, slope variance and rut depth) can be examined. Not well recognized are the facts that the amount of patching at the Road Test was essentially negligeable, and equally, but of great importance *there was no routine maintenance undertaken on the pavement sections* such as might be expected on in-service highway pavements.

After reviewing the data file all observations were removed for sections after overlay. Data from overlaid sections was not used in the original layer coefficient determination. Subsequently, plots were prepared for PSI vs. Traffic, PSI vs. $\log_{10}$ (Traffic) and PSI vs. AASHO day. Examples of these plots are given in Figures 2a, 2b and 2c. Examination of these figures reveals that the AASHO model provides a very poor predictive

model. In addition, both sets of data (observed and predicted) were submitted to analysis through the SPSS statistical package. The "Runs Test" of this statistical package returned a z-statistic of 16, (typically a value of less than 2.0 might be expected). Thus while the overall regression might appear reasonable in terms of the r^2 statistic, the model is otherwise inappropriate, (the values of the r^2 statistic pertaining to the Road Test are not easily found in the literature, however the Authors believe that the values are 0.23 for unweighted traffic and 0.49 for weighted traffic).
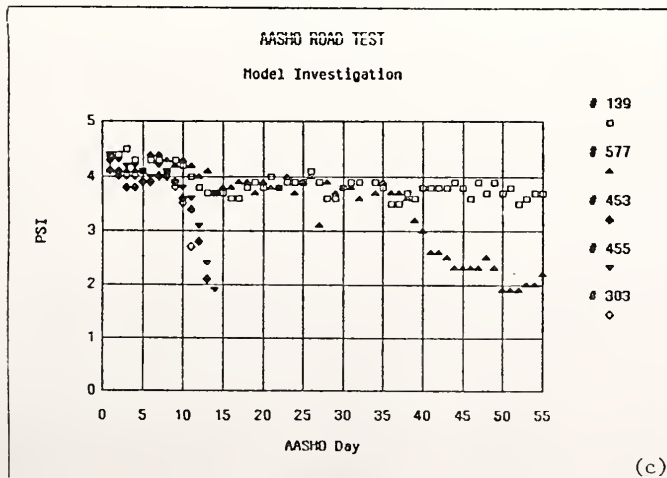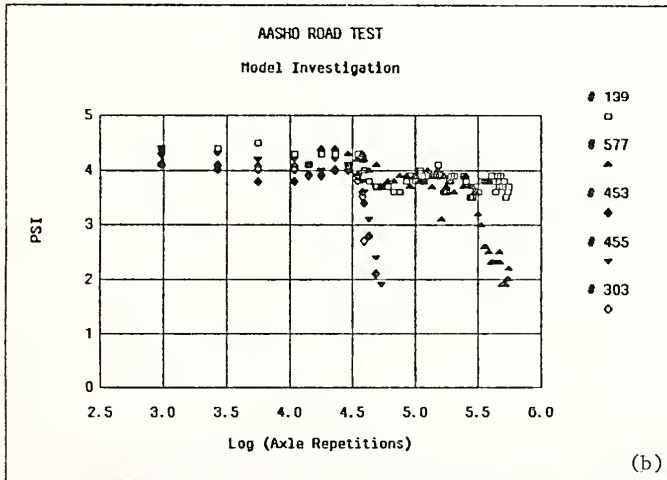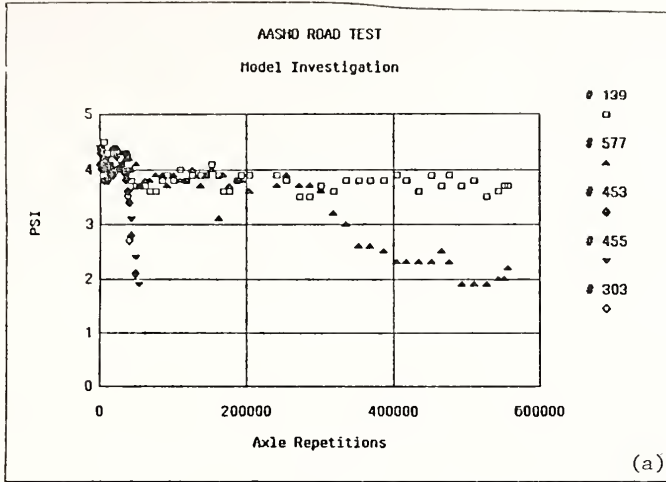
(a)

(b)

Figure 2.   AASHO Road Test
            Serviceability
            Plots

(c)

A review of Figures 2a, 2b and 2c reveals that the shapes of the curves are not as would have been expected. Instead of being smoothly downcurving, the data appeared to turn sharply downward at either one or two reasonably well defined events. This was apparent in both the PSI vs. Traffic and PSI vs. $\log_{10}$ (Traffic) plots, although more pronounced in the former. In the PSI vs. AASHO day plot the locations of these two critical events were particularly noticeable, and were found to correspond very closely with the periods of spring-thaw.

Not all sections showed this tendency; however in those that did not, a downward step was noted occurring at the same times. On the PSI vs. AASHO day plots the curves appeared to be generally piecewise linear, i.e., the PSI decreased linearly at one rate and then linearly at a much increased rate. In some cases, this decrease in PSI took the form of a step function, the original slope being re-established; these observations are shown schematically in Figure 3. Figure 3 oulines three observed performance patterns:

a)   P→A→B,   (rarely observed except on very thick sections and/or very light axle weights), exhibits no significant distress. Figures 2

The slope of the line (rate of decay) is clearly independent of traffic volume (axles/day); it is equally independent of climatic (freeze/thaw) events.

b)   P→A→C→D→E,   a number of sections showed this pattern, where a distinct and sudden loss of serviceability was noted (A→C) coinciding

with the first spring/thaw. Some of these sections cracked but the majority survived the first year uncracked. Those sections which survived cracked failed at the onset of the next winter, while the uncracked survivors generally failed during the following freeze/thaw event.

c)   P→A→E,   a large number of sections exhibited this behaviour, failing rapidly and catastrophically during the first freeze/thaw event.
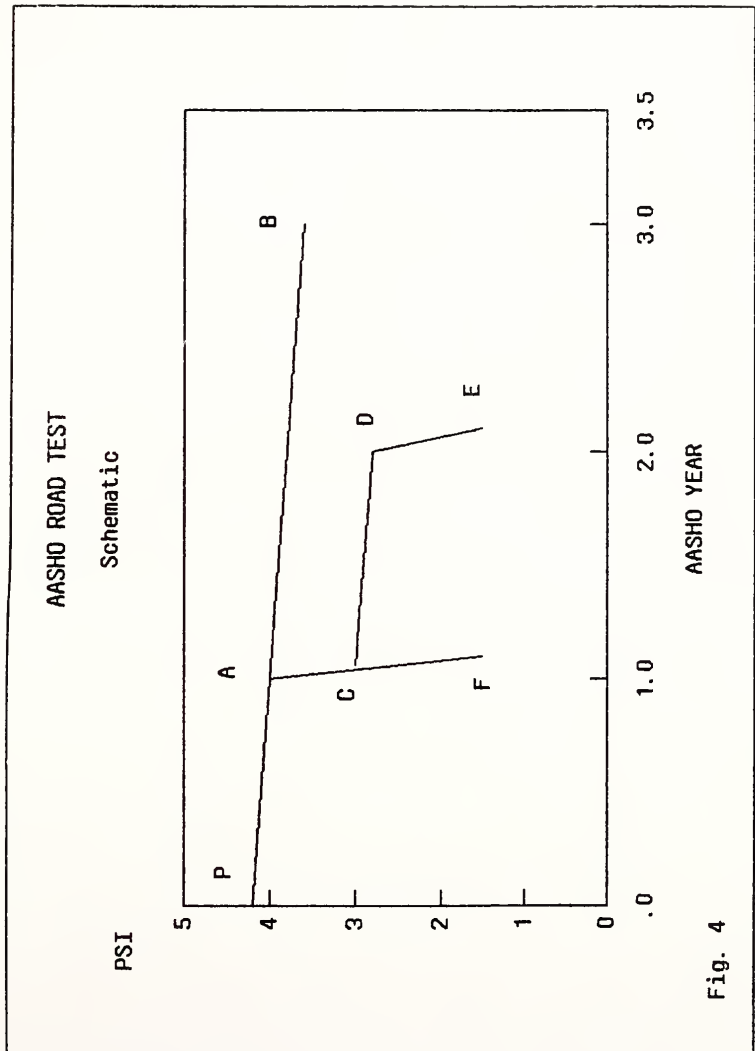
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Figure 3.   AASHO Road Test Performance Schematic

The overall linearity of these plots was particularly puzzling since no suggestion had been made that time was a significant factor, which is implied by this result. In fact, axle repetitions (traffic) were considered the primary forcing variable. A check confirmed that the rate of trafficking (axles per AASHO day) over the period of the Road Test was not constant, however the changes in trafficking rates did not coincide with the changes of slope in the plots. Table 5 shows the rate of section failure ($p_t < 1.5$) is not well correlated with traffic, but is highly correlated with the season of the year. Thus, time as measured by the number of spring/thaw events appears to be of greater significance than previously suspected.

Table 5. AASHO Road Test: traffic and Failure Rates

Season	% Traffic	% Failures
Winter	22	28
Spring	24	60
Summer	30	2
Fall	23	10

In extending the piecewise linear hypothesis from the trafficked sections to the untrafficked sections of Loop 1, only three particularly thin sections displayed the same pattern. As a reult, the Authors concluded that the interaction of time and traffic might be significant. the relatively sudden serviceability loss observed during the spring/thaw periods was addressed by examining the component parts of the PSI. The examination revealed that this phenomenon was paralleled by the initial observation of Class 2 (alligator) and/or Class 3 (granulated) cracking (Figure 4). The full set of observed data from a number of sections was plotted (Figure 5) (instead of only those at $p = 3.5, 3.0, 2.5, 2.0$ and 1.5 as used in the original AASHO Road Test analysis). This full set of data clearly reflects the piecewise linear relationship contrary to the expectation of Eqn (2). The points of intersection of the linear portions of the plots closely match the initial observation of Class 2 and/or Class 3 cracking. In practical terms, two populations of pavement performance have been observed: (a) integral, uncracked and still sealed pavement, and (b) cracked, disintegrating pavements. Since at the Road Test, most Class 2 and/or
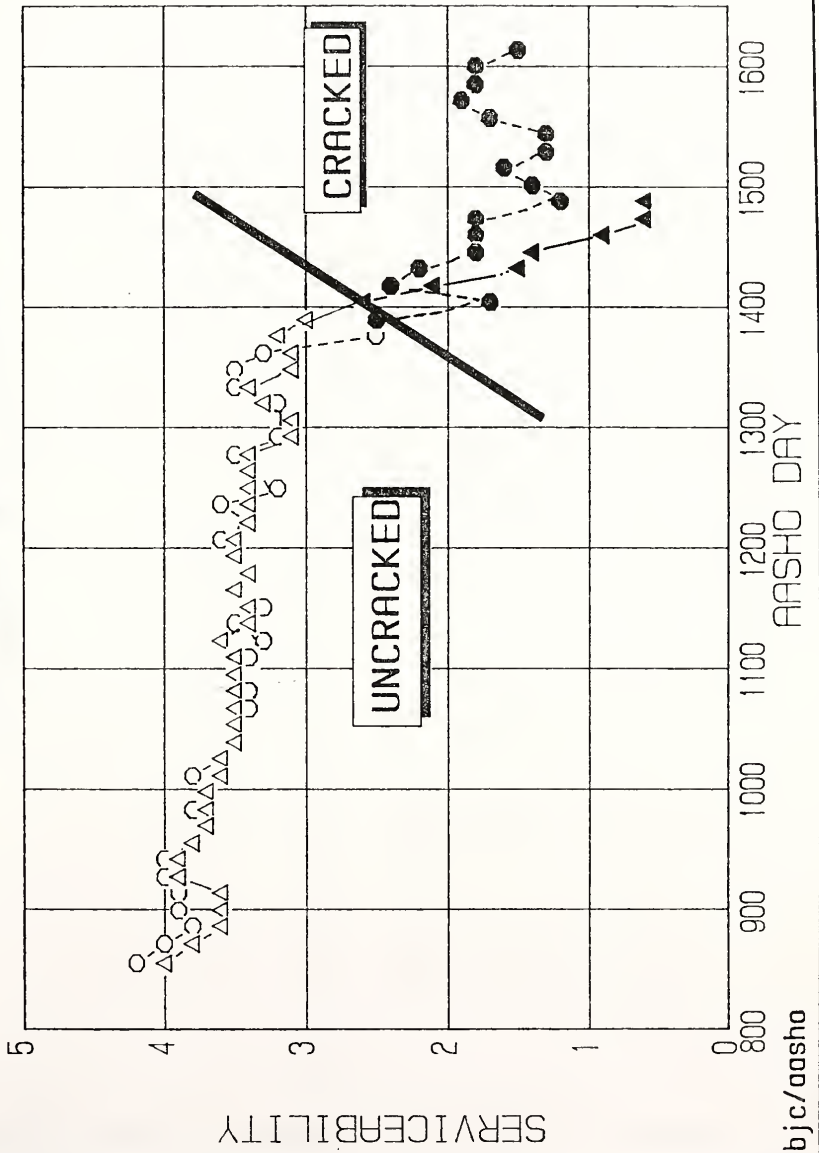
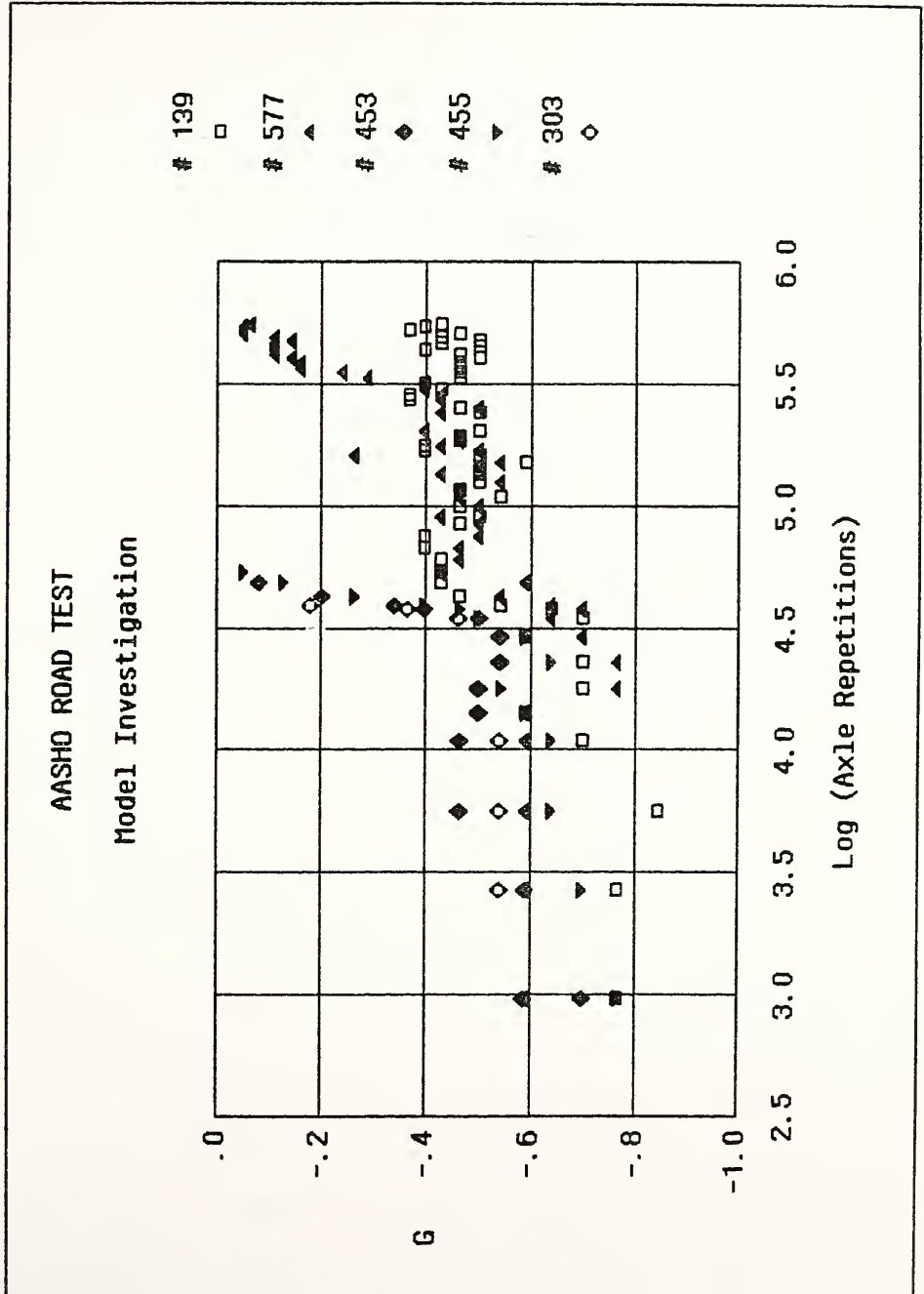Figure 4.   AASHO Road Test Effect of Cracking on Serviceability

Figure 5.   AASHO Road Test Section Serviceability Histories

Class 3 cracking was initially observed at, or about, a PSI of 3.0 to 3.5, it must be stated that the final AASHO equations (4a and 4b) are calibrated to failed, or failing pavements.

5. CUMULATIVE SUM (Cusum) PLOTS

If, using the observed data from any section, the cumulative seviceability loss (ASL) is plotted against AASHO day, a piecewise linear graph will be observed (Fig. 6). (In Figure 6 the serviceability loss is given as $(5.0-p(i))$, and the Cusum as $\sum\limits_{i=1}^{i=t}(5.0-p(i))$, where t is the number of AASHO days from the start of trafficking to the point in time considered). In the following analysis, serviceability loss is defined as $(p_o-p(i))$. Close inspection indicates that the relation rapidly approaches linearity on each leg, i.e., that the data exhibits linearity asymptotically.

If the expected serviceability loss at any time t is represented by a re-arrangement of Equation (1), then:

$$p_o - p(i) = \text{Serviceability loss} = (p_o - p_t)\cdot\left[\frac{W(i)}{\rho}\right]^{\beta} \qquad (5)$$

Then, re-creating the Cusum plot mathematically:

AASHO Day	Incremental Loss	Accumulated Loss
1	$(p_o - p_t)\cdot\left[\frac{W(1)}{\rho}\right]^{\beta}$	$(p_o - p_t)\cdot\left[\frac{W(1)}{\rho}\right]^{\beta}$
2	$(p_o - p_t)\cdot\left[\frac{W(2)}{\rho}\right]^{\beta}$	$(p_o - p_t)\cdot\left[\frac{W(1)^{\beta}+W(2)^{\beta}}{\rho^{\beta}}\right]$
..	..	..
n	$(p_o - p_t)\cdot\left[\frac{W(n)}{\rho}\right]^{\beta}$	$(p_o - p_t)\cdot\left[\frac{W(1)^{\beta}+W(2)^{\beta}+..+W(n)^{\beta}}{\rho^{\beta}}\right]$

In order to approach linearity, $(\text{Cusum}(n+1) - \text{Cusum}(n))$ must approach a constant value in the limit as n increases. Assuming for simplicity that the traffic rate is constant at w axles per AASHO day (i.e., that $W(n)=n\cdot w$), then:

$$\text{Cusum}(n+1) - \text{Cusum}(n) = (p_o - p_t)\cdot\left[\frac{w}{\rho}\right]^{\beta}\cdot n^{\beta} \qquad (6)$$
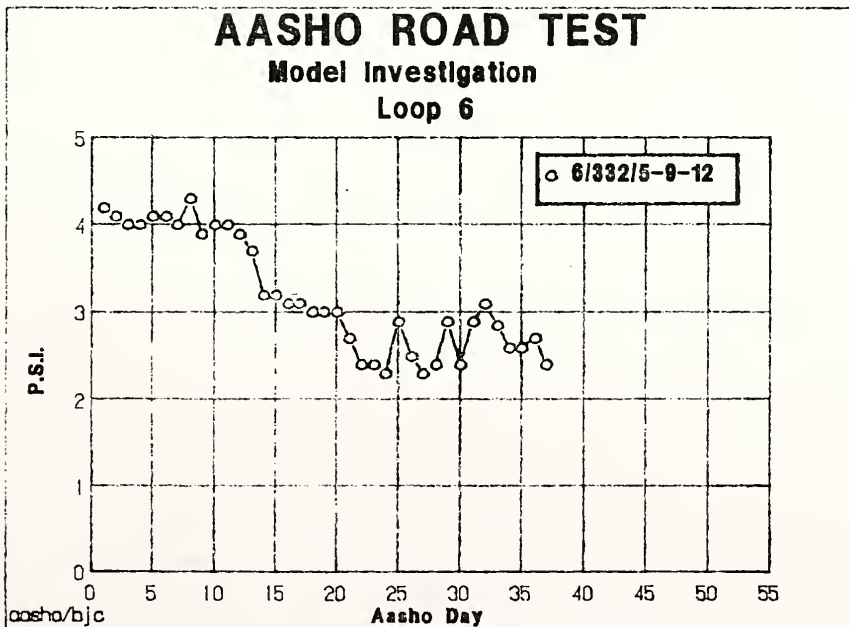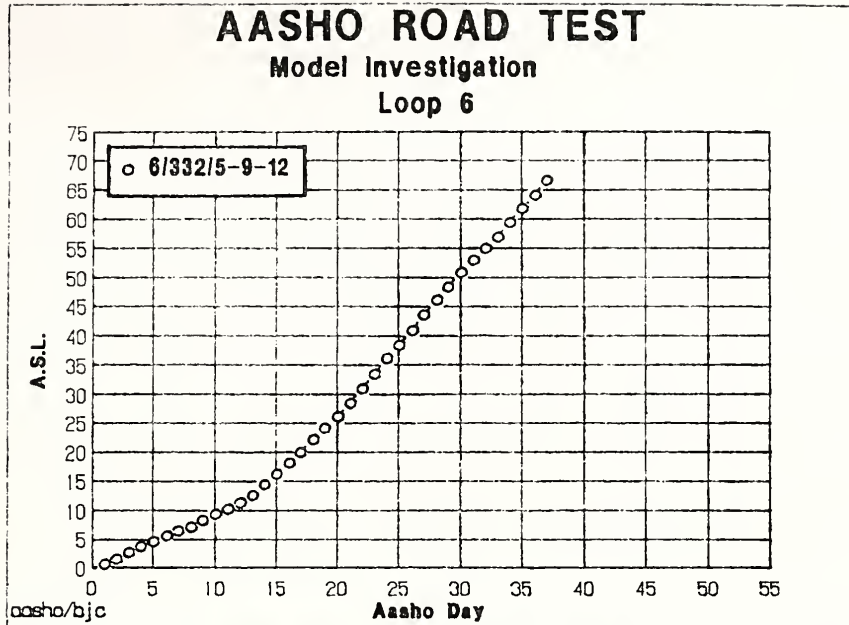
Figure 6. AASHO Road Test Accumulated Serviceability Loss CASR

If this expression is to be constant, then it may be simplified thus:

$$n^{\beta} = \text{constant} \tag{7}$$

This expression is only true in the limit for all n, as n approaches infinity if β is vanishingly small. For a range of structural number, SN, of 1 to 6, the corresponding range of β in Equation (4b) is found to be $0.4 < \beta < 300$. Thus, the piecewise linear relationship observed in Figure (6) *cannot* be supported by the AASHO model.

The implication of this analysis is that on linear portions ($\beta \simeq 0$) of the Cusum plot, traffic (axle weight and repetitions) has no effect, and it is only during those periods of time when the Cusum is in transition from one linear portion to another ($\beta \neq 0$) that traffic has a significant effect on serviceability.

The mathematics of the Cusum transformation have not been developed sufficiently by the Authors to permit the derivation of the relationship which gives rise to the observed asymptotically linear Cusum function.

The smoothing effect of the Cusum transformation can clearly be seen in Figure 6 and was used by the Authors to help identify the critical points (changes of slope) in the behavior of each section.

Figure 7 superimposes the Cusum plots for a number of different sections on the same graph; the axle weight was the same for all of these sections. While not overly evident, it can be seen that (i) until cracking (a change of slope) separates a section from the main plot, the behavior of all of the sections is essentially the same, and (ii) in general, the thicker (total pavement thickness) pavements survive the longest before cracking, and (iii) within pavements of the same or similar total thickness, those with thicker surfacing t_1 survive longer.

6. SURVIVAL PROBABILITY

A probabilistic analysis was made in an attempt to quantify, or model, the step function reported above, which was identified as being triggered, or initiated, by a spring/thaw event. On any given lane of
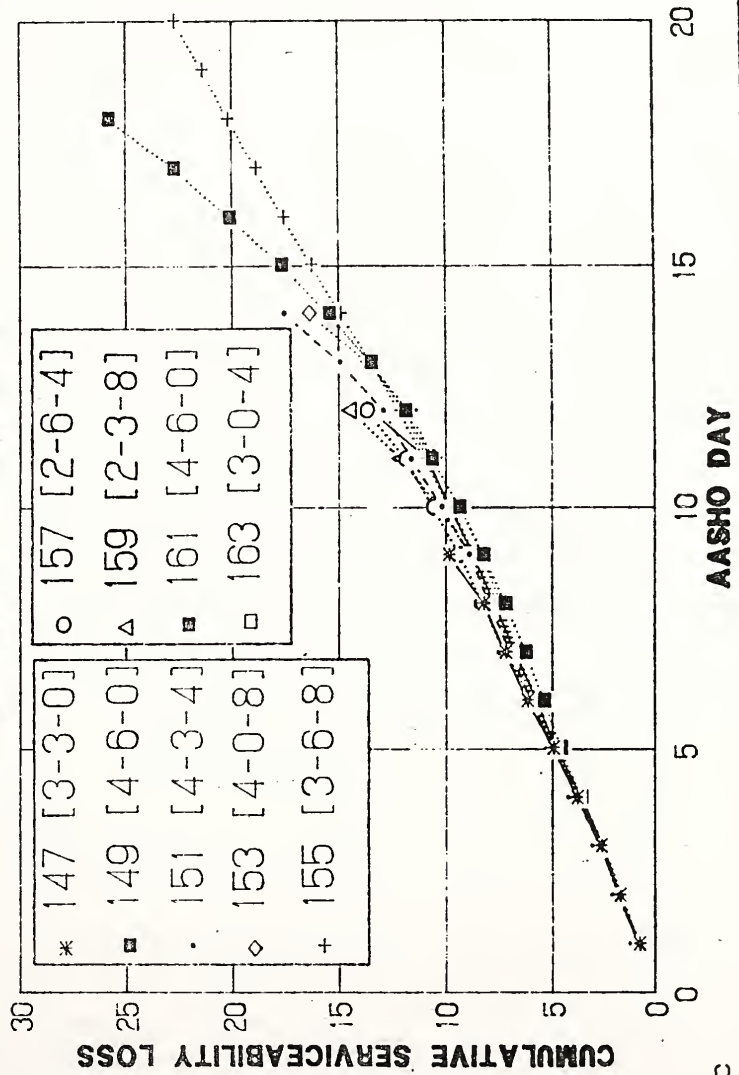
Figure 7.   AASHO Road Test ASL Plots

the Road Test (constant axle weight and type) pavements of various composition either failed or survived the first full seasonal cycle of spring/thaw. By trying to relate the elements of structure to the probability of failure (or survival), certain conclusions could be drawn, i.e., while the thicknesses of each individual layer provided a small measure of correlation with the probability of survival, no factor was found to be as significant as the effect of the total physical thickness (surfacing + base + subbase), regardless of the layer thickness combinations.

While total pavement thickness was particularly significant in this analysis, it is evident that for pavements of the same total thickness, those with thicker surfacing have an enhanced probability of survival. This has been demonstrated by plotting, but the mathematical and statistical analyses are not yet complete.

An example of the first year survival matrix is given in Figure 8 for Lane 1 of Loop 4 (18-kip single axle), where 1 = survival and 0 = failure (did not survive), (the italicized numbers give the full pavement thickness for each section). the composite survival regression curves are given in Figure 9 for all the axle weights and types used in the main factorial experiment.

These observations serve to validate the concept of the US Army Corps of Engineers method for frost design wherein the pavement is first designed from purely structural considerations, and subsequently the total pavement thickness is checked against the anticipated depth of frost penetration.

From the limited data available (two seasonal cycles) the data tends to support the possibility that among survivors, the probability of survival is Markovian, thus if a pavement has a 0.95 probability of first-year survival, then its two-year survival probability is $(0.95)^2$, and its n-year survival probability is $(0.95)^n$.

These curves (Fig. 9) are of course only applicable to the climatic and subgrade conditions of the Road Test. If the subgrade were

AASHO ROAD TEST

1 — Yr Survival Matrix

```
LOOP:.....4...
LANE:......1....
LOAD:..18S...
```

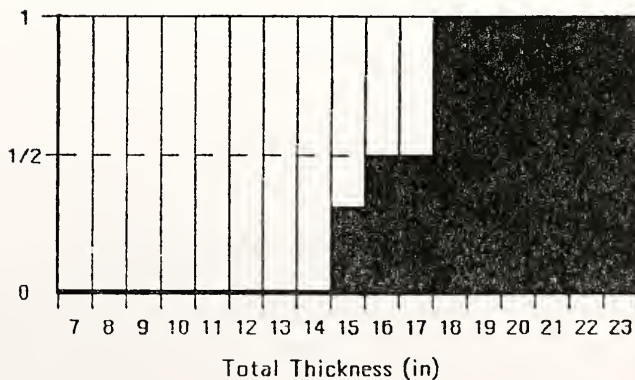| Surfacing	Base		4		8		12	
3	0	0	0	7	0	11	0 0	15
	3	0	0	10	0	14	1	18
	6	0	0	13	0	17	1	21
4	0	0	0	8	0	12	0	16
	3	0	0	11	0 0	15	1	19
	6	0	0	14	1	18	1	22
5	0	0	0	9	0	13	1	17
	3	0	0	12	1	16	1	20
	6	1 1		15	1	19	1	23

Total Thickness (in)

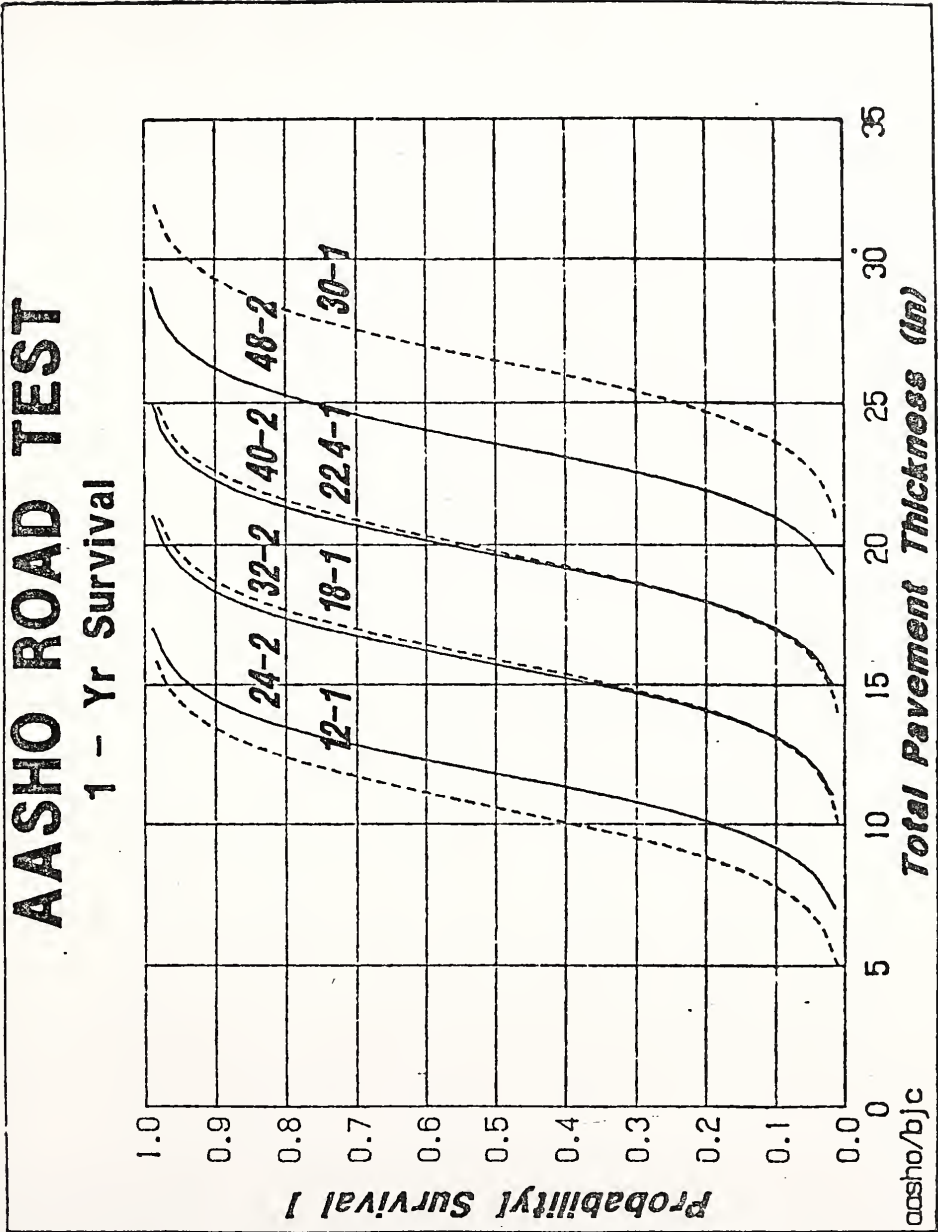Figure 8. AASHO Road Test One-Year Survival Matrix

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Figure 9. AASHO Road Test Survival Probability Plots

free-draining, not frost-susceptible or if there were no frost, then it
might be reasonably expected that these curves would translate
significantly to the left.

7. SUMMARY AND CONCLUSIONS

7.1 AASHO Model

It has been shown that the AASHO model does not represent the observed
behavior of the pavements trafficked at the Road Test. The AASHO model
(Eqn. (1)) is biased to more closely represent the behavior of *cracked*
pavement. Within the AASHO model, the layer coefficients are shown to be
secondary regression coefficients with no direct physical significance.
To attribute to them a significance as indicators of strength is
spurious. Instead, the layer coefficients are indicators of resistance
to serviceability loss.

In the original development of the AASHO model, no cognizance was given
to the effect of climate (including its effect on the subgrade support
characteristics), i.e., the effect of climate was assumed to be constant
from section to section. A tacit assumption was made at the Road Test
that all deterioration in pavement serviceability was due to the
composite effects of traffic (axle weight and frequency) and pavement
structure (materials and layer thicknesses).

However, the present analysis suggests that at Ottawa, Illinois, the
effect of climate was in fact decisive. The initiating event of all
significant deterioration in pavement serviceability was inevitably
linked to spring/thaw. the subsequent performance of the trial sections
was found to be critically dependent upon the observation of Class 2
and/or Class 3 cracking.

The effect of traffic (frequency) is most difficult to define. However,
results of the current analysis indicate that the frequency of loading
is critical *only* during the spring/thaw periods. At other times of the

year, the effect of traffic (axle weight) is explicitly clear in the
AASHO model (Eqns. 3a and 3b), however the effect of the axle weight is

seen to be negligible except in relation to the survival probability, which is significantly effected by the spring/thaw events.

7.2 Alternative Analyses

7.2.1   Cusum analysis.   The Cusum analysis outlined in this paper provides a clear method whereby the performance of Road Test pavements may be analyzed. The surprising smoothing effect and piecewise linearity lend themselves to the identification of changes in performance. It is anticipated that the study of the Cusum plots and their mathematical basis will provide a more defensible foundation from which to build an alternative pavement performance model.

7.2.2   Probabilistic analysis. The variability in the factors and parameters associated with pavement design and performance, however well controlled, lends itself readily to a probabilistic analysis. The first analysis presented here clearly demonstrates the power of such an approach.

The new AASHTO Guide (1986) advocates the use of reliability concepts. the application of the principles of reliability to empirically derivied deterministic formulæ (the AASHO model) is fraught with problems   in implementation and interpretation. The application of probabilistic methods to pavement design would be better served by a *total re-analysis* of the Road Test data (and other data bases) from a probabilistic basis: in this fashion the full model (and its sub-models) wo7\uld be internally consistent and far more transportable.

7.3 Conclusion

The Authors have presented the preliminary results from a study which has highlighted shortcomings in the AASHO model and the interpretation of the Road Test data. As far as is possible, they have sought to provide both observational and mathematical justification for each point raised.

The Authors *strongly* recommend that the AASHO Road Test data be closely scrutinized and re-analyzed in the light of twenty five years of

hindsight, newer pavement technology tools and the more recent concepts of probabilistic analysis and reliability. In this way new (and better) models and sub-models of pavement behavior and performance may be developed.

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APPENDIX B

AASHO ROAD TEST LAYER COEFFICIENT DISTRIBUTIONS

AASHO ROAD TEST LAYER COEFFICIENT

DISTRIBUTIONS

The AASHO model may be stated:

$$\log_{10}(W) = \log_{10}(\rho) + \frac{G}{\beta} \qquad (B1)$$

where   W   =   number of repetitions of axle weight L_1 (kips),
and axle type L_2 (1 = single, 2 = tandem)

   G   = $\log_{10}\left(\dfrac{4.2 - p}{2.7}\right)$ where   p   is   the   present
serviceability index.

ρ, β =   parameters

Based on the AASHO Road Test analysis for weighted traffic, the parameters ρ, β are given by:

$$\beta = 0.4 + \frac{0.081(L_1 + L_2)^{3.23}}{(D + 1)^{5.19} L_2^{3.23}} \qquad (B2)$$

and

$$\rho = \frac{10^{5.93}(D + 1)^{9.36} L_2^{4.33}}{(L_1 + L_2)^{4.79}} \qquad (B3)$$

where   D   =   a thickness index (= $SN = a_1 t_1 + a_2 t_2 + a_3 t_3$)
   a_i =   the layer coefficient of the ith material layer
   t_i =   the thickness of the ith material layer (inches)

Thus, the complete model may be written:

$$\log_{10} W = 5.93 + 9.36 \cdot \log_{10} (D + 1) + 4.33 \cdot \log_{10} (L_2)$$

$$- 4.79 \cdot \log_{10} (L_1 + L_2) + \cfrac{\log_{10}\left(\cfrac{4.2 - p}{2.7}\right)}{0.4 + \cfrac{0.081 (L_1 + L_2)^{3.23}}{(D + 1)^{5.19} L_2^{3.23}}} \qquad (B4)$$

Equation B4 may be re-written:

$$\log_{10} W - 5.93 - 4.33 \cdot \log_{10} (L_2) + 4.79 \cdot \log_{10} (L_1 + L_2) = (LHS) =$$

$$(RHS) = \cfrac{\log_{10}\left(\cfrac{4.2 - p}{2.7}\right)}{0.4 + \cfrac{0.081 (L_1 + L_2)^{3.23}}{(D + 1)^{5.19} L_2^{3.23}}} - 9.36 \cdot \log_{10} (D + 1) \qquad (B5)$$

using the values of $\log_{10} W$ and p tabulated for each section in HRB SR 61E: Report 5, Appendix A, pp 244 – 248, the expression in equation B5 above may be solved for D by a non-linear least-squares regression analysis (e.g., IMSL routine ZXSSQ). Thus for each section, an estimate of the thickness index (D), as well as the individual layer thicknesses (t_i) are known.

A linear regression (SPSS routine REGRESSION) of the estimated thickness index (D), or the Structural Number (SN = D), against $a_1 t_1 + a_2 t_2 + a_3 t_3$ (where the t_i are known) for the entire 284 sections was then performed. This yielded the estimates of the layer coefficients, a_i, and provided estimates of the standard deviations $\sigma(a_i)$.

APPENDIX C

ROSENBLUETH POINT ESTIMATE METHOD

ROSENBLUETH POINT ESTIMATE METHOD

Conventionally, a distribution is represented by the *mean* value (μ) and the *standard deviation* (σ). However, if the distribution is not symmetrical, or departs significantly from the normal, or gaussian, two further parameters may be used; the *skewness* (β_1) and the *kurtosis* (β_2).

These measures of distribution arise from the various axiomatic definitions of probability measure. Thus:

$$\int_{-\infty}^{+\infty} f(x) \cdot dx = 1 = \sum_{1}^{n} p_i = 1 \qquad (C1)$$

Equation C1 states that the total probability of an event is unity, or **Prob**[the event will occur] + **Prob**[the event will not occur] = 1. This definition is of primary axiomatic importance in porbability.

$$\int_{-\infty}^{+\infty} x \cdot f(x) \cdot dx = \mu = \sum_{1}^{n} p_i \cdot x \qquad (C2)$$

Equation C2 defines the *mean*, or *expectation*, of the distribution, and provide a measure of location. If only a single measure is available to describe the distribution, the mean, μ, provides that measure.

$$\int_{-\infty}^{+\infty} (x-\mu)^2 \cdot f(x) \cdot dx = \sigma^2 = \sum_{1}^{n} p_i \cdot (x-\mu)^2 \qquad (C3)$$

The *variance*, defined in equation C3 provides a measure of the dispersion of the distribution about the mean value. The square root of the variance is termed the *standard deviation*. A true Gaussian distribution is fully defined by the definitions C1 thru C3.

Distributions which exhibit skew, or asymmetry, require a further descriptive statistic, which normalized is given by:

$$\int_{-\infty}^{+\infty} (x-\mu)^3 \cdot f(x) \cdot dx = \beta_1 \cdot \sigma^3 = \sum_{1}^{n} p_i \cdot (x-\mu)^3 \qquad (C4)$$

The parameter β_1 is called the *skew*, and is a measure of the degree of asymmetry of the distribution. If $\beta_1 = 0$ the distribution is symmetrical; if $\beta_1 > 0$. the distribution is skwewed right (i.e.. long tail to the right of the mean). and if $\beta_1 < 0$, the distribution is skewed left (long tail to the left of the mean).

$$\int_{-\infty}^{+\infty} (x-\mu)^4 \cdot f(x) \cdot dx = \beta_2 \cdot \sigma^4 = \sum_{1}^{n} p_i \cdot (x-\mu)^4 \qquad (C5)$$

Similarly, the parameter β_2 is a measure of the *flatness*, or *peakedness* of the distribution. and is termed the coefficient of kurtosis. If $\beta_2 < 3$, then the distribution is flatter than the gaussian distribution, and if $\beta_2 > 3$ the distribution is more peaked than the gaussian.

The distribution parameters defined above are sufficient to define most, if not all, practical (unimodal) distributions. However, in practical engineering terms, only the mean, μ, standard deviation, σ, and very occasionally, the skew, β_1, are known.

If these parameters are known for a variable, say x, (i.e., $\mu(x)$, $\sigma(x)$, $\beta_1(x)$ and $\beta_2(x)$), and it is required to estimate the distribution parameters of a *function of x*, $f(x)$, then conventionally, recourse must be had to a truncated Taylor series expansion of the function. This method is termed the First Order Second Moment method (FOSM). For a function of a single variable, this results in:

$$E[f(x)] = \mu(x) + \frac{1}{2} \cdot \frac{\partial^2 f(x)}{\partial x^2} \cdot \sigma^2(x) \qquad (C6)$$

and

$$\sigma^2(f(x)) = \left[\frac{\partial f(x)}{\partial x}\right]^2 \cdot \sigma(x) + \frac{\beta_2(x)-1}{4} \cdot \left[\frac{\partial^2 f(x)}{\partial x^2}\right]^2 \cdot \sigma^2(x) +$$

$$+ \beta_1(x) \cdot \sigma^3(x) \cdot \left[\frac{\partial f(x)}{\partial x} \cdot \frac{\partial^2 f(x)}{\partial x^2}\right] \qquad (C7)$$

Thus it is seen that to make use of the FOSM technique, not only must the distribution parameters be known, but also the first and second derivatives of the function. This is not always easy or feasible.

With a multivariate function, say $f(x,y)$, not only must the distribution parameters be known for both x and y, but also the first and second derivatives of the function with respect to both x and y, and the covariance $COV(x,y)$ or the correlation coefficient $\rho_{x,y}$. The resulting equations needed to define the mean and variance of the function $f(x,y)$ are not trivial.

The Rosenblueth Point Estimate Method (PEM) provides a convenient and computationally attractive method to achieve the same end results. This method accounts for correlations between variables with little or no added computational difficulty.

Rosenblueth drew on the analogy between probability distributions and the distributed vertical loading on a rigid, simply-supported horizontal beam. The magnitude of the "load" is defined by equation C1 above, and the center of gravity, centroid or location of the resultant, of the loading diagram, is located by using equation C2 above. Equations C3 thru C5 may be recognized as forms of a general relationship:

$$\int_{-\infty}^{+\infty} (x-\mu)^n \cdot f(x) \cdot dx = \beta_{n-2} \cdot \sigma^n = \sum_1^m p_i \cdot (x_i - \mu)^n \qquad (C8)$$

when $n=2$, the term $\beta_{n-2} \cdot \sigma^n$ (or variance σ^2) represents the radius of gyration of the loading diagram about the resultant. Other values

of n lead to representations of the nth moment about the mean. or resultant.

Since the "beam" is simply supported, the the following relationships may be stated (using the discrete forms of C1 - C4):

$$p_+ + p_- = 1 \tag{C9a}$$

$$p_+ \cdot x_+ + p_- \cdot x_- = \mu(x) \tag{C9b}$$

$$p_+ \cdot (x_+ - \mu(x))^2 + p_- \cdot (x_- - \mu(x))^2 = \sigma^2(x) \tag{C9c}$$

$$p_+ \cdot (x_+ - \mu(x))^3 + p_- \cdot (x_- - \mu(x))^3 = \beta_1(x) \cdot \sigma^3(x) \tag{C9d}$$

where p_+ and p_- represent either the magnitudes of the "beam" reactions in the beam analogy, or the two point-estimates of probability, and x_+ and x_- represent the reaction locations.

Solving the above equations for p_+, p_-, x_+ and x_-, the following relationships are found:

$$p_+ = \frac{1}{2} \cdot \left[1 \pm \sqrt{1 - \frac{1}{1 + \left[\frac{\beta_1(x)}{2}\right]^2}} \right] \tag{C10a}$$

[NB. The sign used ($\pm$) is the opposite to that of $\beta_1(x)$]

$$p_- = 1 - p_+ \tag{C10b}$$

$$x_+ = \mu(x) + \sigma(x) \cdot \sqrt{p_-/p_+} \tag{C10c}$$

$$x_- = \mu(x) - \sigma(x) \cdot \sqrt{p_+/p_-} \tag{C10d}$$

Note that if the distribution of x is symmetrical ($\beta_1(x) = 0$), then these become:

$$p_+ = p_- = \frac{1}{2} \tag{C11a}$$

$$x_+ = \mu(x) + \sigma(x) \tag{C11b}$$

$$x_- = \mu(x) - \sigma(x) \tag{C11c}$$

It should be noted that no information has been lost. If the distribution is characterized by the mean and variance, then two 'pieces' of information are available; two 'pieces' of information remain after the above transformation, i.e., (p_+,x_+) and (p_-,x_-).

Example 1

Given that $\mu(x) = 35°$ and $\sigma(x) = 5°$, what is the distribution of $\phi(x) = \sin(x)$?

Given only this information, we may assume that $\beta_1(x) = 0$, then $p_+ = p_- = 0.5$, and $x_+ = 35° + 5° = 40°$, and $x_- = 35° - 5° = 30°$. So that:

$$\phi_+ = \phi(x_+) = \sin(40°) = 0.6428$$
$$\phi_- = \phi(x_-) = \sin(30°) = 0.5000$$
$$E[\phi] = \mu(\phi) = p_+\phi_+ + p_-\phi_- = 0.5\cdot(0.6428 + 0.5000) = 0.5714$$
$$E[\phi^2] = p_+\phi_+^2 + p_-\phi_-^2 = 0.5\cdot(0.6428^2 + 0.5000^2) = 0.3316$$
$$\sigma^2(\phi) = E[\phi^2] - E[\phi]^2 = 0.3316 - (0.5714)^2 = 0.0051$$
$$\sigma(\phi = 0.0714$$

Thus the distribution parameters for the function $\phi = f(x)$ have been found, i.e., $\mu\phi) = 0.5714$, and $\sigma(\phi) = 0.0714$.

Example 2

Given that $\mu(x) = 35°$ and $\sigma(x) = 5°$ and $\beta_1(x) = 1$, what is the distribution of $\phi(x) = \sin(x)$?

Given this information, then by C10a: $p_+ = 0.2764$ and $p_- = 0.7236$ and $x_+ = 35° + 8.09° = 43.09°$ and $x_- = 35° - 3.09° = 31.91°$. So that:

$$\phi_+ = f(\phi_+) = \sin(43.09°) = 0.6831$$
$$\phi_- = f(\phi_-) = \sin(31.91°) = 0.5286$$

$$E[\phi] = \mu(\phi) = p_+\phi_+ + p_-\phi_- = 0.2764\cdot0.6831 + 0.7236\cdot0.5286 = 0.5713$$
$$E[\phi^2] = p_+\phi_+^2 + p_-\phi_-^2 = 0.2764\cdot0.6831^2 + 0.7236\cdot0.5286^2 = 0.3312$$
$$\sigma^2(\phi) = E[\phi^2] - E[\phi]^2 = 0.3312 - (0.5713)^2 = 0.0048$$
$$\sigma(\phi = 0.0694$$

Again, the distribution paramaeters for the function of the asymmetrical variable have been found: $\mu(\phi) = 0.5713$ and $\sigma(\phi) = 0.0694$.

An extension of the same principles leads to the solution of the case of multivariate functions, say $z = f(x,y)$. As before, the

individual point estimates (i.e., x_+, x_-, y_+ and y_-) are found using equations C10 above. The the relationship may be written:

$$E[z^n] = p_{++} z^n_{++} + p_{+-} z^n_{+-} + p_{-+} z^n_{-+} + p_{--} z^n_{--}$$

where, in this case, $z_{++} = f(x_+, y_+)$, and $p_{++} = p_{--} = \dfrac{1 + \rho_{x,y}}{4}$ and $p_{+-} = p_{-+} = \dfrac{1 - \rho_{x,y}}{4}$, where $\rho_{x,y}$ is the correlation coefficient between x and y.

If the dependent variable is a function of n variables, then 2^n values are required to provide the distribution parameters, and the correlated $p_{\pm\pm\pm}$ values are found as follows (here n=3):

$$P_{+++} = p_{---} = \frac{(1 + \rho_{12} + \rho_{23} + \rho_{31})}{2^3}$$

$$p_{++-} = p_{--+} = \frac{(1 + \rho_{12} - \rho_{23} - \rho_{31})}{2^3}$$

$$p_{+-+} = p_{-+-} = \frac{(1 - \rho_{12} - \rho_{23} + \rho_{31})}{2^3}$$

$$p_{+--} = p_{-++} = \frac{(1 - \rho_{12} + \rho_{23} - \rho_{31})}{2^3}$$

The above is a very simple outline of the Rosenblueth Point Estimate Method. A more complete development is given in "Reliability-Based Design in Civil Engineering", by M. E. Harr, McGraw-Hill Book Company, 1987.

EXAMPLE from PROJECT

Equation 19 gives the Ullidtz equation of the van der Poel relationship for binder stiffness, S_b, which is given as:

$$S_b = 1.157 \times 10^{-7} \times t^{-0.368} \times e^{-PI} \times (T_{RB} - T)^5$$

where S_b is the Binder Stiffness (MPa)

           t is the time of loading (sec)

PI   is the Penetration Index

T_{RB} is the Ring and Ball Softening Point ($^{\circ}$C)

T   is the material temperature ($^{\circ}$C)

From Table 17, the IDOH time of loading (All Highways) is given as 0.0108 seconds, with a standard deviation of 0.0009 seconds.

The Penetration Index, PI, and Ring and Ball Softening Point, T_{RB}, may be found from equations 20 thru 22, which are functions of the initial penetration, pen_i. From Table 15, the distribution parameters of the initial penetration are given as: $E[pen_i] = 64$, and $\sigma[pen_i] = 4$ and $\beta_1[pen_i] = 0.39$.

Using the pavement temperature distribution parameters (All State) from table 20, i.e., $E[T] = 12.47$ C and $\sigma[T] = 12.12$ C.

The two variables, time of loading, t, and pavement temperature, T, are assumed symmetrical (since skewness, β_1, is zero), and may be reduced to their x_+ and x_- values, thus:

$$t_+ = E[t] + \sigma[t] = 0.0108 + 0.0009 = 0.0117$$
$$t_- = E[t] - \sigma[t] = 0.0108 - 0.0009 = 0.0099$$

and

$$T_+ = E[T] + \sigma[T] = 12.47 + 12.12 = 24.59$$
$$T_- = E[T] - \sigma[T] = 12.47 - 12.12 = 0.35$$

Since the initial penetration, pen_i, has an element of skew ($\beta_1 = 0.39$), the procedure is as follows:

$$p_+ = \frac{1}{2} \cdot \left[1 - \sqrt{1 - \frac{1}{1 + \left[\frac{0.39}{2}\right]^2}} \right] = 0.4043$$

and

$$p_- = 1 - p_+ = 1 - 0.4043 = 0.5957$$

so that

$$pen_{i+} = E[pen_i] + \sigma[pen_i] \cdot \sqrt{p_-/p_+}$$

or

$$pen_{i+} = 64 + 4 \cdot \sqrt{0.5957/0.4043} = 68.86$$

and

$$pen_{i-} = 64 - 4 \cdot \sqrt{0.4043/0.5957} = 60.70$$

Since the penetration index, PI, and the softening point, T_{RB}, are functions of initial penetration, pen_i, the following may be found by direct substitution of the above results in Equations 20 thru 22, thus:

$$pen_{i+} = 68.86 \quad pen_{i+} = 44.76 \quad T_{RB+} = 55.63 \quad PI_{+} = -0.1435$$

and

$$pen_{i-} = 60.70 \quad pen_{i-} = 39.46 \quad T_{RB-} = 57.07 \quad PI_{-} = -0.1221$$

Now, following the Rosenblueth PEM method above:(all S_b are in MPa)

$$S_{b+++} = 1.157 \times 10^{-7}(0.0117)^{-0.368} \cdot e^{0.1435}(55.63 - 24.59)^{5} = 19.5775$$
$$S_{b++-} = 1.157 \times 10^{-7}(0.0117)^{-0.368} \cdot e^{0.1435}(55.63 - 0.35)^{5} = 354.3255$$
$$S_{b+-+} = 1.157 \times 10^{-7}(0.0117)^{-0.368} \cdot e^{0.1221}(57.07 - 24.59)^{5} = 24.2856$$
$$S_{b+--} = 1.157 \times 10^{-7}(0.0117)^{-0.368} \cdot e^{0.1221}(57.07 - 0.35)^{5} = 394.4114$$
$$S_{b-++} = 1.157 \times 10^{-7}(0.0099)^{-0.368} \cdot e^{0.1435}(55.63 - 24.59)^{5} = 21.0315$$
$$S_{b-+-} = 1.157 \times 10^{-7}(0.0099)^{-0.368} \cdot e^{0.1435}(55.63 - 0.35)^{5} = 376.7914$$
$$S_{b--+} = 1.157 \times 10^{-7}(0.0099)^{-0.368} \cdot e^{0.1221}(57.07 - 24.59)^{5} = 25.8254$$
$$S_{b---} = 1.157 \times 10^{-7}(0.0099)^{-0.368} \cdot e^{0.1221}(57.07 - 0.35)^{5} = 419.4190$$

Thus:

	S_b	S_b^2	$Log(S_b)$	$(Log(S_b))^2$
+++	19.5775	383.28	1.2918	1.6686
++-	354.3255	125546.56	2.5494	6.4995
+-+	24.2856	589.79	1.3853	1.9192
+--	394.4114	155560.35	2.5959	6.7390
-++	21.0315	442.32	1.3229	1.7500
-+-	376.7914	141971.76	2.5761	6.6363
--+	25.8254	666.95	1.4120	1.9939
---	419.4190	175912.30	2.6226	6.8783
	1635.6673	601073.31	15.7560	34.0848

From which:

$$E[S_b] = 204.5 \text{ MPa } (1.4 \text{ psi}) \qquad \sigma[S_b] = 184.6 \text{ MPa } (1.27 \text{ psi})$$

or

$$E[Log(S_b)] = 1.9695 \qquad \sigma[Log(S_b)] = 0.6178$$

Two facts lead to the choice of the logarithmic solution, (i) equations 24a and 24b require the use of $Log(S_b)$ rather than S_b, and (ii) use of the arithmetic option presumes that S_b can be negative ($\mu - 3\sigma = -349$ MPa), while the logarithmic option prevents this absurdity!

This latter example of the Point Estimate method is only one small part of the full analysis, but it clearly demonstrates the power of the method, and highlights the overall computational simplicity when used with an intrinsically non-linear multivariate expression.